Checkpoint: Assess Your Understanding, pages 602–604

7.1

1. **Multiple Choice** How many roots does the equation \( \sin 6x = \frac{1}{3} \) have over the domain \( 0 \leq x < 2\pi \)?

   A. 2  
   B. 4  
   C. 6  
   D. 12  

2. Use graphing technology to solve each equation over the given domain. Give the roots to the nearest hundredth.

   a) \( 1 + 2 \sin x = 1 - 3 \cos x; 0 \leq x \leq 2\pi \)

      Graph the corresponding function: \( y = 2 \sin x + 3 \cos x \)
      Determine the approximate zeros in the given domain.
      The roots are approximately: \( x = 2.16 \) and \( x = 5.30 \)
      Substitute each root into the given equation to verify.

   b) \( 2 = \cos x + 2 \cos^2 x; -2\pi \leq x \leq 2\pi \)

      Graph the corresponding function: \( y = \cos x + 2 \cos^2 x - 2 \)
      Determine the approximate zeros in the given domain.
      The roots are approximately: \( x = \pm 0.67 \) and \( x = \pm 5.61 \)
      Substitute each root into the given equation to verify.
3. Use graphing technology to determine the general solution of each equation over the set of real numbers. Give the answers to the nearest hundredth.

a) \[4 \tan \frac{x}{5} = 0\]

Graph the corresponding function: \[y = 4 \tan x - 5\]
The period of the function is \(\pi\).
Determine the zero in the domain \(0 < x < \pi\).
The root is approximately: \(x = 0.90\)
The general solution is approximately: \(x = 0.90 + \pi k, k \in \mathbb{Z}\)

b) \[6 \cos^2 x + \cos x = 1\]

Graph the corresponding function: \[y = 6 \cos^2 x + \cos x - 1\]
The period of the function is \(2\pi\).
Determine the zeros in the domain \(0 < x < 2\pi\).
The roots are approximately: \(x = 1.23, x = 2.09, x = 4.19, x = 5.05\)
The general solution is approximately: \(x = 1.23 + 2\pi k, k \in \mathbb{Z}\) or \(x = 2.09 + 2\pi k, k \in \mathbb{Z}\) or \(x = 4.19 + 2\pi k, k \in \mathbb{Z}\) or \(x = 5.05 + 2\pi k, k \in \mathbb{Z}\)

7.2

4. Multiple Choice Which number is a root of the equation \[3 \sin x + 1 = 5 \sin x - 1\] over the domain \(0 \leq x < 2\pi\)?

A. 0  B. \(\pi\)  C. \(\frac{\pi}{2}\)  D. \(\frac{3\pi}{2}\)

5. Use algebra to solve the equation \(\sqrt{2} \cos 2x + 1 = 0\) over the domain \(-\pi < x < \pi\), then write the general solution of the equation.

\[\sqrt{2} \cos 2x = -1\]
\[\cos 2x = -\frac{1}{\sqrt{2}}\]
The terminal arm of angle \(2x\) lies in Quadrant 2 or 3.
The reference angle for angle \(2x\) is: \(\cos^{-1} \left( \frac{1}{\sqrt{2}} \right) = \frac{\pi}{4}\)

In Quadrant 2, \(2x = \frac{3\pi}{4}\)
\[x = \frac{3\pi}{8}\]

In Quadrant 3, \(2x = -\frac{3\pi}{4}\)
\[x = -\frac{3\pi}{8}\]

The period of \(\cos 2x\) is \(\pi\), so other roots are:
\[x = \frac{3\pi}{8} - \pi\] and \[x = -\frac{3\pi}{8} - \pi\]
\[x = -\frac{5\pi}{8}\] and \[x = \frac{5\pi}{8}\]
The roots are: \(x = \pm \frac{3\pi}{8}\) and \(x = \pm \frac{5\pi}{8}\)
The general solution is: \(x = \frac{3\pi}{8} + \pi k, k \in \mathbb{Z}\) or \(x = \frac{5\pi}{8} + \pi k, k \in \mathbb{Z}\)
6. Verify that \( \frac{\pi}{6} \) and \( \frac{5\pi}{6} \) are two roots of the equation \( 4 \cos^2 x - 3 = 0 \).

Substitute each given value in the equation.

For \( x = \frac{\pi}{6} \):

\[
\text{L.S.} = 4 \cos^2 \left( \frac{\pi}{6} \right) - 3 \\
= 4 \left( \frac{\sqrt{3}}{2} \right)^2 - 3 \\
= 0 \\
= \text{R.S.}
\]

For \( x = \frac{5\pi}{6} \):

\[
\text{L.S.} = 4 \cos^2 \left( \frac{5\pi}{6} \right) - 3 \\
= 4 \left( -\frac{\sqrt{3}}{2} \right)^2 - 3 \\
= 0 \\
= \text{R.S.}
\]

For each value of \( x \), the left side is equal to the right side, so the roots are verified.

7. Use algebra to solve the equation \( 10 \sin^2 x + 11 \sin x = -3 \) over the domain \( 90^\circ \leq x \leq 360^\circ \). Give the roots to the nearest degree.

\[
10 \sin^2 x + 11 \sin x + 3 = 0 \\
(2 \sin x + 1)(5 \sin x + 3) = 0
\]

Either \( 2 \sin x + 1 = 0 \) or \( 5 \sin x + 3 = 0 \)

\[
\sin x = -0.5 \\
\sin x = -0.6
\]

The reference angle is: \( \sin^{-1}(0.5) = 30^\circ \)  
The reference angle is: \( \sin^{-1}(0.6) \approx 37^\circ \)

The terminal arm of angle \( x \) lies in Quadrant 3 or 4. 

In Quadrant 3, \( x = 180^\circ + 30^\circ \), or \( 210^\circ \) 
In Quadrant 4, \( x = 360^\circ - 30^\circ \), or \( 330^\circ \)

The roots are: \( x = 210^\circ \), \( x \approx 217^\circ \), \( x \approx 323^\circ \), \( x = 330^\circ \)