1. Determine the value of each trigonometric ratio. Use exact values where possible; otherwise write the value to the nearest thousandth.

a) \( \tan \left( \frac{\pi}{4} \right) \)

\[ = -1 \]

b) \( \cos 600^\circ \)

\[ \begin{align*} & = \cos 240^\circ \\ & = -\frac{1}{2} \end{align*} \]

c) \( \sec ( -210^\circ ) \)

\[ \begin{align*} & = \cos ( -210^\circ ) \\ & = \frac{1}{\cos 150^\circ} \\ & = -\frac{2}{\sqrt{3}} \end{align*} \]

d) \( \sin 765^\circ \)

\[ \begin{align*} & = \sin 45^\circ \\ & = \frac{1}{\sqrt{2}} \end{align*} \]

e) \( \cot 21^\circ \)

\[ \begin{align*} & = \frac{1}{\tan 21^\circ} \\ & \approx 2.605 \end{align*} \]

f) \( \csc 318^\circ \)

\[ \begin{align*} & = \frac{1}{\sin 318^\circ} \\ & \approx -1.494 \]

2. To the nearest degree, determine all possible values of \( \theta \) for which \( \cos \theta = 0.76 \), when \( -360^\circ \leq \theta \leq 360^\circ \).

Since \( \cos \theta \) is positive, the terminal arm of angle \( \theta \) lies in Quadrant 1 or 4.

The reference angle is: \( \cos^{-1}(0.76) \approx 41^\circ \)

For the domain \( 0^\circ \leq \theta \leq 360^\circ \):

- In Quadrant 1, \( \theta = 41^\circ \)
- In Quadrant 4, \( \theta = 360^\circ - 41^\circ \), or approximately 319°

For the domain \( -360^\circ \leq \theta \leq 0^\circ \):

- In Quadrant 1, \( \theta = -360^\circ + 41^\circ \), or approximately -319°
- In Quadrant 4, \( \theta = -41^\circ \)

3. As a fraction of \( \pi \), determine the length of the arc that subtends a central angle of 225° in a circle with radius 3 units.

Arc length: \( \frac{225}{360}(2\pi)(3) = \frac{15}{4}\pi \)

4. a) Convert each angle to degrees. Give the answer to the nearest degree where necessary.

i) \( \frac{5\pi}{3} \)

\[ \begin{align*} & = \frac{5(180^\circ)}{3} \\ & = 300^\circ \]

ii) \( -\frac{10\pi}{7} \)

\[ \begin{align*} & = -\frac{10(180^\circ)}{7} \\ & = -257^\circ \]

iii) 4

\[ \begin{align*} & = 4 \left( \frac{180^\circ}{\pi} \right) \\ & = 229^\circ \]
b) Convert each angle to radians.
   i) 150°  
   \[150° = 150 \left( \frac{\pi}{180} \right) = \frac{5\pi}{6}\]  
   ii) \(-240°\)  
   \[\text{No conversion necessary as it is already in radians.}\]  
   iii) 485°  
   \[485° = 485 \left( \frac{\pi}{180} \right) = \frac{97\pi}{36}\]

5. In a circle with radius 5 cm, an arc of length 6 cm subtends a central angle. What is the measure of this angle in radians, and to the nearest degree?

   Angle measure is: \[\frac{\text{arc length}}{\text{radius}} = \frac{6}{5}\]
   \[\text{In degrees, } 1.2 = 1.2 \left( \frac{180°}{\pi} \right) = 69°\]
   The angle measure is 1.2 radians or approximately 69°.

6. A race car is travelling around a circular track at an average speed of 120 km/h. The track has a diameter of 1 km. Visualize a line segment joining the race car to the centre of the track. Through what angle, in radians, will the segment have rotated in 10 s?

   In 1 s, the car travels: \[\frac{120}{60} \cdot 60 = \frac{1}{3} \text{ km}\]
   So, in 10 s, the car travels: \[10 \cdot \frac{1}{3} = \frac{10}{3} \text{ km}\]

   Angle measure is: \[\frac{\text{arc length}}{\text{radius}} = \frac{\frac{1}{3}}{1} = \frac{1}{3}\]

   In 10 s, the segment will have rotated through an angle of \(\frac{1}{3}\) radian.

7. Determine the value of each trigonometric ratio. Use exact values where possible; otherwise write the value to the nearest thousandth.

   a) \(\sin \left( \frac{\pi}{4} \right)\)  
   \[= \frac{\sqrt{3}}{2}\]  
   b) \(\cos \left( \frac{5\pi}{6} \right)\)  
   \[= -\frac{\sqrt{3}}{2}\]  
   c) \(\sec \left( -\frac{\pi}{2} \right)\)  
   \[= \frac{1}{\cos \left( -\frac{\pi}{2} \right)}\]
   which is undefined

   d) \(\tan \left( \frac{15\pi}{4} \right)\)  
   \[= \tan \left( \frac{\pi}{4} \right) = 1\]  
   e) \(\csc 5\)  
   \[= \frac{1}{\sin 5} \approx -1.043\]  
   f) \(\cot \left( -22.8 \right)\)
   \[= \frac{1}{\tan \left( -22.8 \right)} \approx -0.954\]
8. P(3, -1) is a terminal point of angle \( \theta \) in standard position.

a) Determine the exact values of all the trigonometric ratios for \( \theta \).

Let the distance between the origin and \( P \) be \( r \).

Use: \( x^2 + y^2 = r^2 \)

Substitute: \( x = 3, y = -1 \)

\[
r = \sqrt{x^2 + y^2} = \sqrt{3^2 + (-1)^2} = \sqrt{10}
\]

\[
\sin \theta = -\frac{1}{\sqrt{10}} \quad \csc \theta = -\sqrt{10} \quad \cos \theta = \frac{3}{\sqrt{10}}
\]

\[
\sec \theta = \frac{\sqrt{10}}{3} \quad \tan \theta = -\frac{1}{3} \quad \cot \theta = -3
\]

b) To the nearest tenth of a radian, determine possible values of \( \theta \) in the domain \(-2\pi \leq \theta \leq 2\pi\).

The terminal arm of angle \( \theta \) lies in Quadrant 4.

The reference angle is: \( \tan^{-1}(\frac{1}{3}) = 0.3217\ldots \)

So, \( \theta = -0.3217\ldots \)

The angle between 0 and \( 2\pi \) that is coterminal with \(-0.3217\ldots \) is:

\[
2\pi - 0.3217\ldots = 5.9614\ldots
\]

Possible values of \( \theta \) are approximately: 6.0 and -0.3

9. Use graphing technology to graph each function below for 

\(-2\pi \leq x \leq 2\pi\), then list these characteristics of the graph: amplitude, period, zeros, domain, range, and the equations of the asymptotes.

a) \( y = \sin x \)

The amplitude is 1. The period is \( 2\pi \). The zeros are 0, \( \pm \pi \), \( \pm 2\pi \). The domain is \(-2\pi \leq x \leq 2\pi \). The range is \(-1 \leq y \leq 1\). There are no asymptotes.

b) \( y = \cos x \)

The amplitude is 1. The period is \( 2\pi \). The zeros are \( \pm \frac{\pi}{2} \), \( \pm \frac{3\pi}{2} \).

The domain is \(-2\pi \leq x \leq 2\pi \). The range is \(-1 \leq y \leq 1\). There are no asymptotes.

c) \( y = \tan x \)

There is no amplitude. The period is \( \pi \). The zeros are 0, \( \pm \pi \), \( \pm 2\pi \).

The domain is \( x \neq \pm \frac{\pi}{2} \), \( \pm \frac{3\pi}{2} \). The range is \( y \in \mathbb{R} \). The equations of the asymptotes are \( x = \pm \frac{\pi}{2} \) and \( x = \pm \frac{3\pi}{2} \).
6.5

10. On the same grid, sketch graphs of the functions in each pair for \(0 \leq x \leq 2\pi\), then describe your strategy.

a) \(y = \sin x\) and \(y = \sin \left(x - \frac{\pi}{4}\right)\)

For the graph of \(y = \sin x\), I used the completed table of values from Lesson 6.4.
The horizontal scale is 1 square to \(\frac{\pi}{4}\) units, because the phase shift is \(\frac{\pi}{4}\).
I then shifted several points \(\frac{\pi}{4}\) units right and joined the points to get the graph of \(y = \sin \left(x - \frac{\pi}{4}\right)\).

b) \(y = \cos x\) and \(y = \frac{3}{2} \cos x\)

For the graph of \(y = \cos x\), I used the completed table of values from Lesson 6.4.
I multiplied every \(y\)-coordinate by 1.5, plotted the new points, then joined them to get the graph of \(y = \frac{3}{2} \cos x\).
6.6

11. a) Graph \( y = \frac{1}{2} \sin \left( x + \frac{\pi}{6} \right) + 2 \) for \(-2\pi \leq x \leq 2\pi\).

Explain your strategy.

Sample response: I graphed \( y = \frac{1}{2} \sin 3x \), shifted several points \( \frac{\pi}{6} \) units left and 2 units up, then joined the points to get the graph of 
\( y = \frac{1}{2} \sin \left( x + \frac{\pi}{6} \right) + 2 \).

b) List the characteristics of the graph you drew.

- The amplitude is \( \frac{1}{2} \).
- The period is \( \frac{2\pi}{3} \).
- There are no zeros.
- The domain is \(-2\pi \leq x \leq 2\pi \).
- The range is \( \frac{3}{2} \leq y \leq \frac{5}{2} \).

12. An equation of the function graphed below has the form
\( y = a \cos b(x - c) + d \). Identify the values of \( a, b, c, \) and \( d \) in the equation, then write an equation for the function.

Sample response: The equation of the centre line is \( y = \frac{1}{2} \), so the vertical translation is \( \frac{1}{2} \) unit up and \( d = \frac{1}{2} \).

The amplitude is: \( \frac{3 - (-2)}{2} = \frac{5}{2} \), so \( a = \frac{5}{2} \).

Choose the \( x \)-coordinates of two adjacent maximum points, \(-\frac{\pi}{3}\) and \( \frac{11\pi}{3} \). The period is: \( \frac{11\pi}{3} - \left( -\frac{\pi}{3} \right) = 4\pi \)

So, \( b \) is: \( \frac{2\pi}{4\pi} = \frac{1}{2} \)

To the left of the \( y \)-axis, the cosine function begins its cycle at 
\( x = -\frac{\pi}{3} \), so a possible phase shift is \( -\frac{\pi}{3} \), and \( c = -\frac{\pi}{3} \).

Substitute for \( a, b, c, \) and \( d \) in: \( y = a \cos b(x - c) + d \)

An equation is: \( y = \frac{5}{2} \cos \left( \frac{1}{2} \left( x + \frac{\pi}{3} \right) \right) + \frac{1}{2} \)
13. A water wheel has diameter 10 m and completes 4 revolutions each minute. The axle of the wheel is 8 m above a river.

a) The wheel is at rest at time \( t = 0 \) s, with point P at the lowest point on the wheel. Determine a function that models the height of P above the river, \( h \) metres, at any time \( t \) seconds. Explain how the characteristics of the graph relate to the given information.

The time for 1 revolution is 15 s.
At \( t = 0 \), \( h = 3 \)
At \( t = 7.5 \), \( h = 13 \)
The graph begins at (0, 3), which is a minimum point.
The first maximum point is at (7.5, 13).
The next minimum point is after 1 cycle and it has coordinates (15, 3).
The position of the first maximum is known, so use a cosine function:
\[ h(t) = a \cos \left( \frac{2\pi}{b} (t - c) \right) + d \]
The constant in the equation of the centre line of the graph is the height of the axle above the river, so its equation is: \( h = 8 \); and this is also the vertical translation, so \( d = 8 \)
The amplitude is one-half the diameter of the wheel, so \( a = 5 \)
The period is the time for 1 revolution, so \( b = \frac{15}{15} \)
A possible phase shift is: \( c = 7.5 \)
An equation is: \( h(t) = 5 \cos \left( \frac{2\pi}{15}(t - 7.5) \right) + 8 \)

b) Use technology to graph the function. Use this graph to determine:

i) the height of P after 35 s

Graph: \( Y = 5 \cos \left( \frac{2\pi}{15}(X - 7.5) \right) + 8 \)
Determine the Y-value when \( X = 35 \).
After 35 s, P is 10.5 m high.
ii) the times, to the nearest tenth of a second, in the first 15 s of motion that P is 11 m above the river

Graph: $Y = 5 \cos \frac{2\pi}{15}(X - 7.5) + 8$ and $Y = 11$

Determine the Y-coordinates of the first two points of intersection.
P is 11 m above the river after approximately 5.3 s and 9.7 s.