Lesson 6.5 Exercises, pages 521–526

A

3. Identify the indicated characteristic of each function.
   a) amplitude of \( y = 5 \sin x \)
      The amplitude is 5.
   b) amplitude of \( y = 2 \cos x \)
      The amplitude is 2.
   c) period of \( y = \sin 10x \)
      The period is \( \frac{2\pi}{10} = \frac{\pi}{5} \)
   d) period of \( y = \tan 4x \)
      The period is \( \frac{\pi}{4} \)
   e) phase shift of \( y = \sin \left( x - \frac{\pi}{7} \right) \)
      The phase shift is \( \frac{\pi}{7} \)
   f) phase shift of \( y = \cos \left( x + \frac{\pi}{12} \right) \)
      The phase shift is \( -\frac{\pi}{12} \)

B

4. For each function below, sketch the graph for \( -\pi \leq x \leq \pi \), then identify each characteristic:
   i) amplitude
   ii) period
   iii) zeros
   iv) equations of any asymptotes
   v) domain of the function
   vi) range of the function
   a) \( y = \cos x \)
      i) The amplitude is 1.    ii) The period is 2\( \pi \).
      iii) The zeros are \( \pm \frac{\pi}{2} \).    iv) There are no asymptotes.
      v) The domain is \( x \in \mathbb{R} \).    vi) The range is \( -1 \leq y \leq 1 \).
b) \( y = \sin x \)

![Graph of \( y = \sin x \)](image)

i) The amplitude is 1.  
ii) The period is \( 2\pi \).
iii) The zeros are 0, \( \pm \pi \).  
iv) There are no asymptotes.
v) The domain is \( x \in \mathbb{R} \).  
vi) The range is \(-1 \leq y \leq 1\).

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5. Does this graph represent a periodic function? Explain.

![Graph of \( y = f(x) \)](image)

No, the graph does not represent a periodic function because the graph does not repeat in regular intervals.
6. Use technology.

   a) i) Graph each function.
       \( y = 2 \cos x \quad y = -3 \cos x \quad y = \frac{1}{3} \cos x \)

   ii) How does varying the value of \( a \) affect the graph of \( y = a \cos x \)?

      When \( a = 1 \), the graph is \( y = \cos x \) with amplitude 1. As \( a \) varies,
      the amplitude varies. When \( a > 1 \), the graph of \( y = \cos x \) is
      stretched vertically by a factor of \( a \) and the amplitude increases;
      when \( 0 < a < 1 \), the graph of \( y = \cos x \) is compressed vertically by
      a factor of \( a \) and the amplitude decreases; when \( a < 0 \), the graph is
      also reflected in the \( x \)-axis.

   b) i) Graph each function.
       \( y = \sin 3x \quad y = \sin (-4x) \quad y = \sin \frac{3}{4}x \)

   ii) How does varying the value of \( b \) affect the graph of \( y = \sin bx \)?

      When \( b = 1 \), the graph is \( y = \sin x \) and its period is \( 2\pi \). As \( b \) varies,
      the period of the graph varies. When \( b > 1 \), the graph of \( y = \sin x \) is
      compressed horizontally by a factor of \( \frac{1}{b} \) and the period decreases;
      when \( 0 < b < 1 \), the graph of \( y = \sin x \) is stretched horizontally by
      a factor of \( \frac{1}{b} \) and the period increases; when \( b < 0 \), the graph is also
      reflected in the \( y \)-axis.

   c) i) Graph each function.
       \( y = \cos \left(x - \frac{\pi}{6}\right) \quad y = \cos \left(x - \frac{\pi}{4}\right) \quad y = \cos \left(x + \frac{\pi}{3}\right) \)

   ii) How does varying the value of \( c \) affect the graph of
       \( y = \cos (x - c) \)?

      When \( c = 1 \), the graph is \( y = \cos x \) with phase shift 0. As \( c \) varies, the
      phase shift varies. When \( c > 0 \), the graph of \( y = \cos x \) is translated
      \( c \) units right; when \( c < 0 \), the graph is translated \( c \) units left.

   d) i) Graph each function.
       \( y = \sin x + 1 \quad y = \sin x - 2 \quad y = \sin x + 0.5 \)

   ii) How does varying the value of \( d \) affect the graph of
       \( y = \sin x + d \)?

      When \( d = 0 \), the graph is \( y = \sin x \). As \( d \) varies, the graph of \( y = \sin x \)
      is translated vertically. When \( d > 0 \), the graph is translated \( d \) units up;
      when \( d < 0 \), the graph is translated \( d \) units down.
7. Sketch the graph of each function. Describe your strategy.

a) \( y = \cos x + 1 \)

I used the completed table of values for \( y = \cos x \) from Lesson 6.4, translated each point 1 unit up, extended the pattern, then drew a smooth curve through the points.

b) \( y = \sin 2x \)

I used the completed table of values for \( y = \sin x \) from Lesson 6.4, halved each \( x \)-coordinate, extended the pattern, then drew a smooth curve through the points.

c) \( y = \cos \left( x - \frac{\pi}{3} \right) \)

I used the completed table of values for \( y = \cos x \) from Lesson 6.4, translated each point \( \frac{\pi}{3} \) units right, extended the pattern, then drew a smooth curve through the points.
d) \( y = 2 \sin x \)

\[ y = 2 \sin x \]

I used the completed table of values for \( y = \sin x \) from Lesson 6.4, doubled each \( y \)-coordinate, extended the pattern, then drew a smooth curve through the points.

8. Use technology to graph \( y = \sin \left( x + \frac{\pi}{2} \right) \) and \( y = \cos x \).

Explain the result.

The graphs coincide. The graph of \( y = \cos x \) is the image of the graph of \( y = \sin x \) after a horizontal translation of \( \frac{\pi}{2} \) units left; that is, for any angle \( x \) radians, \( \cos x = \sin \left( x + \frac{\pi}{2} \right) \).

9. A student says that the amplitude of this sinusoidal function is 5.

Is the student correct? Explain.

No, the amplitude is one-half of the vertical distance between a maximum point and a minimum point, which is 2.

10. Sketch the graph of each function. Identify its characteristics.

a) \( y = \csc x \)

Take the reciprocal of each \( x \)-value in the completed table for \( y = \sin x \) in Lesson 6.4, plot the points, extend the pattern, then join the points with 2 smooth curves. There is no amplitude. The period is \( 2\pi \). There are no zeros. The equations of the asymptotes are \( x = k\pi, k \in \mathbb{Z} \). The domain is \( x \neq k\pi, k \in \mathbb{Z} \). The range is \( y \geq 1 \) or \( y \leq -1 \).
b) \( y = \cot x \)

Take the reciprocal of each \( x \)-value in the completed table for \( y = \tan x \) in Lesson 6.4, plot the points, extend the pattern, then join the points with 2 smooth curves. There is no amplitude. The period is \( \pi \). The zeros are \( (2k + 1)\frac{\pi}{2}, k \in \mathbb{Z} \). The equations of the asymptotes are \( x = k\pi, k \in \mathbb{Z} \). The domain is \( x \neq k\pi, k \in \mathbb{Z} \). The range is \( y \in \mathbb{R} \).

11. Use technology. Graph the function \( y = \sin x + \cos x \).

The function is periodic because its values repeat at regular intervals.
The function is sinusoidal because its maximum and minimum values are equidistant from the centre line.