1. Sketch a graph of each absolute function. Identify the intercepts, domain, and range.
   a) $y = |-4x + 2|$  
   b) $y = |(x + 4)(x - 2)|$

   Draw the graph of $y = -4x + 2$. It has $x$-intercept 0.5. Reflect, in the $x$-axis, the part of the graph that is below the $x$-axis. From the graph, the $x$-intercept is 0.5, the $y$-intercept is 2, the domain of $y = |-4x + 2|$ is $x \in \mathbb{R}$, and the range is $y \geq 0$.

   Draw the graph of $y = (x + 4)(x - 2)$. It has $x$-intercepts $-4$ and 2. The axis of symmetry is $x = -1$, so the vertex is at $(-1, -9)$. Reflect, in the $x$-axis, the part of the graph that is below the $x$-axis. From the graph, the $x$-intercepts are $-4$ and 2, the $y$-intercept is 8, the domain of $y = |(x + 4)(x - 2)|$ is $x \in \mathbb{R}$, and the range is $y \geq 0$.

2. Write each absolute value function in piecewise notation.
   a) $y = |-x - 9|$
   
   $y = -x - 9$ when $-x - 9 \geq 0$
   $-x - 9 \geq 0$
   $-x \geq 9$
   $x \leq -9$
   $y = (-x - 9)$,
   or $y = x + 9$ when $-x - 9 < 0$
   $-x < 9$
   $x > -9$
   So, using piecewise notation:
   $$y = \begin{cases} 
   -x - 9, & \text{if } x \leq -9 \\
   x + 9, & \text{if } x > -9
   \end{cases}$$

   b) $y = |2x(x + 5)|$

   The $x$-intercepts of the graph of $y = 2x(x + 5)$ are $x = 0$ and $x = -5$. The graph opens up, so between the $x$-intercepts, the graph is above the $x$-axis. For the graph of $y = |2x(x + 5)|$:
   For $x \leq -5$ or $x \geq 0$, the value of $2x(x + 5) \geq 0$
   For $-5 < x < 0$, the value of $2x(x + 5) < 0$; that is, $y = -2x(x + 5)$.
   So, using piecewise notation:
   $$y = \begin{cases} 
   2x(x + 5), & \text{if } x \leq -5 \text{ or } x \geq 0 \\
   -2x(x + 5), & \text{if } -5 < x < 0
   \end{cases}$$
3. Solve by graphing.
   a) \(3 = |-2x + 4|\)

   To graph \(y = |2x + 4|\), graph \(y = -2x + 4\), then reflect, in the \(x\)-axis, the part of the graph that is below the \(x\)-axis. The line \(y = 3\) intersects \(y = |2x + 4|\) at (0.5, 3) and (3.5, 3). So, the solutions are \(x = 0.5\) and \(x = 3.5\).

   b) \(|x^2 - 5x| = 6\)

   Enter \(y = |x^2 - 5x|\) and \(y = 6\) in the graphing calculator.
   The line \(y = 6\) appears to intersect \(y = |x^2 - 5x|\) at 4 points: \((-1, 6), (2, 6), (3, 6),\) and \((6, 6)\). So, the equation has 4 solutions: \(x = -1, x = 2, x = 3,\) and \(x = 6\).

4. Use algebra to solve each equation.
   a) \(2 = |(x - 1)^2 - 2|\)

   When \((x - 1)^2 - 2 \geq 0:\)
   \[2 = (x - 1)^2 - 2\]
   \[4 = (x - 1)^2\]
   \[x = 3\] or \(x = -1\)

   So, \(x = -1, x = 1,\) and \(x = 3\) are the solutions.

   When \((x - 1)^2 - 2 < 0:\)
   \[2 = -((x - 1)^2 - 2)\]
   \[-2 = (x - 1)^2 - 2\]
   \[0 = (x - 1)^2\]
   \[x = 1\]

   b) \(2x = \frac{1}{2}|3x - 5|\)

   \[4x = |3x - 5|\]
   \[4x = 3x - 5\]
   \[4x = -(3x - 5)\]

   if \(3x - 5 \geq 0\)
   That is, if \(x \geq \frac{5}{3}\)
   When \(x \geq \frac{5}{3}\)

   \[x = 5\]
   \[x = \frac{5}{3}\]

   \(-5\) is not greater than or equal to \(\frac{5}{3}\); so \(-5\) is not a solution.

   \(\frac{5}{3} < \frac{5}{2}\) so \(\frac{5}{7}\) is a root.
   The solution is \(x = \frac{5}{7}\).
5. Identify the equation of the vertical asymptote of the graph of 
\[ y = \frac{1}{-2x + 5}, \] then graph the function.

The graph of \( y = -2x + 5 \) has slope \(-2\), x-intercept \(\frac{5}{2}\), and y-intercept 5.
The graph of \( y = \frac{1}{-2x + 5} \) has a horizontal asymptote \( y = 0 \) and a vertical asymptote \( x = \frac{5}{2} \). Points \((3, -1)\) and \((2, 1)\) are common to both graphs. Some points on \( y = -2x + 5 \) are \((1, 3), (0, 5), (4, -3),\) and \((5, -5)\).
So, points on \( y = \frac{1}{-2x + 5} \) are \((1, \frac{1}{3}), (0, 0.2), (4, -\frac{1}{3}),\) and \((5, -0.2)\).

6. Use the graph of \( y = f(x) \) to sketch a graph of \( y = \frac{1}{f(x)} \).

Identify the equations of the asymptotes of the graph of each reciprocal function.

a) 
Horizontal asymptote: \( y = 0 \)
x-intercept is \(\frac{1}{2}\), so vertical asymptote is \( x = \frac{1}{2} \). Points \((0, 1)\) and \((1, -1)\) are common to both graphs. Some points on \( y = f(x) \) are \((-1, 3)\) and \((2, -3)\). So, points on \( y = \frac{1}{f(x)} \) are \((-1, \frac{1}{3})\) and \((2, -\frac{1}{3})\).

b) 
Horizontal asymptote: \( y = 0 \)
x-intercept is 6, so vertical asymptote is \( x = 6 \). Points \((7, 1)\) and \((5, -1)\) are common to both graphs. Some points on \( y = f(x) \) are \((8, 2)\) and \((4, -2)\). So, points on \( y = \frac{1}{f(x)} \) are \((8, 0.5)\) and \((4, -0.5)\).
7. Use the graph of \( y = \frac{1}{f(x)} \) to graph the linear function \( y = f(x) \).

Describe your strategy.

[Graph of \( y = \frac{1}{f(x)} \) and \( y = f(x) \)]

Vertical asymptote is \( x = 1.5 \), so graph of \( y = f(x) \) has \( x \)-intercept 1.5. Mark points at \( y = 1 \) and \( y = -1 \) on graph of \( y = \frac{1}{f(x)} \), then draw a line through these points for the graph of \( y = f(x) \).

8. Use a graphing calculator or graphing software.

For which values of \( q \) does the graph of
\[
y' = \frac{1}{-(x - 3)^2 + q}
\]

have:

a) no vertical asymptotes?

Look at the quadratic function \( y = -(x - 3)^2 + q \). The graph opens down. For the graph of its reciprocal function to have no vertical asymptotes, the quadratic function must have no \( x \)-intercepts. So, the vertex of the quadratic function must be below the \( x \)-axis; that is, \( q < 0 \).

b) one vertical asymptote?

Look at the quadratic function \( y = -(x - 3)^2 + q \). The graph opens down. For the graph of its reciprocal function to have one vertical asymptote, the quadratic function must have one \( x \)-intercept. So, the vertex of the quadratic function must be on the \( x \)-axis; that is, \( q = 0 \).

c) two vertical asymptotes?

Look at the quadratic function \( y = -(x - 3)^2 + q \). The graph opens down. For the graph of its reciprocal function to have two vertical asymptotes, the quadratic function must have two \( x \)-intercepts. So, the vertex of the quadratic function must be above the \( x \)-axis; that is, \( q > 0 \).
9. Determine the equations of the vertical asymptotes of the graph of each reciprocal function.

Graph to check the equations.

a) \( y = \frac{1}{(x - 2)^2 - 9} \)

\((x - 2)^2 - 9 = 0\) when
\((x - 2)^2 = 9\); that is, when
\(x = 5\) or \(x = -1\). So, the
graph of \( y = \frac{1}{(x - 2)^2 - 9} \) has
2 vertical asymptotes, \(x = 5\)
and \(x = -1\). I used my graphing
calculator to show that my
equations are correct.

b) \( y = \frac{1}{-(x - 2)^2 - 9} \)

\(-(x - 2)^2 - 9 = 0\) when
\((x - 2)^2 = -9\). Since the square
of a number is never negative,
the graph of \( y = -(x - 2)^2 - 9 \)
has no \(x\)-intercepts, and the
graph of \( y = \frac{1}{-(x - 2)^2 - 9} \) has
no vertical asymptotes. I used my
graphing calculator to show that
my equations are correct.

8.5

10. On the graph of each quadratic function \( y = f(x) \), sketch a graph
of the reciprocal function \( y = \frac{1}{f(x)} \). Identify the vertical asymptotes,
if they exist.

a) The graph of \( y = f(x) \) has no
\(x\)-intercepts, so the graph of
\( y = \frac{1}{f(x)} \) has no vertical
asymptotes and has Shape 1.
Horizontal asymptote: \( y = 0 \)
Points \((2, -4), (3, -2),\) and
\((4, -4)\) lie on \( y = f(x) \), so
points \((2, -0.25), (3, -0.5),\)
and \((4, -0.25)\) lie on
\( y = \frac{1}{f(x)} \).

b) The graph of \( y = f(x) \) has 1
\(x\)-intercept, so the graph of
\( y = \frac{1}{f(x)} \) has 1 vertical asymptote,
\(x = 5\), and has Shape 2.
Horizontal asymptote: \( y = 0 \)
Plot points where the line \( y = 1 \)
intersects the graph of \( y = f(x) \).
These points are common to both
graphs.
11. On the graph of each reciprocal function \( y = \frac{1}{f(x)} \), sketch a graph of the quadratic function \( y = f(x) \).

a) The graph has one vertical asymptote, \( x = 5 \), so the graph of \( y = f(x) \) has vertex (5, 0). The line \( y = -1 \) intersects the graph at 2 points that are common to both graphs.

b) The graph has 2 vertical asymptotes, so the graph of \( y = f(x) \) has 2 \( x \)-intercepts. Plot points where the asymptotes intersect the \( x \)-axis. The point (0, -1) is on the line of symmetry, so (0, -1) is the vertex of \( y = f(x) \).