Lesson 7.1 Exercises, pages 527–531

4. Which expressions are rational expressions? Justify your choices.
   a) \( \frac{x + 3}{5} \)     b) \( \frac{\sqrt{x} + 2}{4x} \)     c) \( \frac{9}{7} \) 
   d) \( \frac{3x + 9}{x^2 - 2} \)     e) \( \frac{3\sqrt[3]{b} + 2b}{16 - b^2} \)     f) \( \frac{x^2 + 2x - 7}{x + 3} \)

Parts a, c, and f are rational expressions because each expression is the quotient of two polynomials. Parts b and e are not rational expressions because they each contain the root of a variable; part d is not a rational expression because it has a variable as an exponent.

5. Identify the non-permissible values of the variable for each rational expression.
   a) \( \frac{2 - x^2}{x + 5} \) 
      \[ x + 5 = 0 \]
      \[ x = -5 \]
      So, \( x = -5 \) is the non-permissible value.
   b) \( \frac{x + 1}{(x - 2)(x + 8)} \) 
      \[ (x - 2)(x + 8) = 0 \]
      \[ x - 2 = 0 \] or \( x + 8 = 0 \)
      \[ x = 2 \] or \( x = -8 \)
      So, \( x = 2 \) and \( x = -8 \) are the non-permissible values.

6. Determine whether the given value of \( x \) is a non-permissible value for the rational expression. Explain how you know.
   a) \( \frac{3x + 9}{(x - 5)(x + 6)} \); \( x = 6 \) 
      No, \( x = 6 \) does not result in the denominator equal to 0.
   b) \( \frac{5x}{x^2 - 4} \); \( x = -2 \) 
      Yes, \( x = -2 \) results in the denominator equal to 0.
7. Simplify each rational expression. State the non-permissible values of the variables.

a) \( \frac{25mn}{5m} \)

The non-permissible value is: \( m = 0 \)

\[ \frac{25mn}{5m} = \frac{5n}{1}m \]

\( m \neq 0 \)

b) \( \frac{2x(x - 3)}{x - 3} \)

The non-permissible value is: \( x = 3 \)

\[ \frac{2x(x - 3)}{x - 3} = \frac{2x(x - 3)}{x - 3} \]

\( x \neq 3 \)

c) \( \frac{(x - 3)(x + 4)}{(x + 4)(x + 6)} \)

The non-permissible values are: \( x = -4 \) and \( x = -6 \)

\[ \frac{(x - 3)(x + 4)}{(x + 4)(x + 6)} = \frac{(x - 3)(x + 4)}{(x + 4)(x + 6)} \]

\( x \neq -6, -4 \)

d) \( \frac{3x}{12x(x + 5)} \)

The non-permissible values are: \( x = 0 \) and \( x = -5 \)

\[ \frac{3x}{12x(x + 5)} = \frac{3x}{12x(x + 5)} \]

\( x \neq -5, 0 \)

B

8. Determine the non-permissible values for each rational expression.

a) \( \frac{x^2 + 3}{x^2 - x - 20} \)

\[ x^2 - x - 20 = 0 \]

\[ (x - 5)(x + 4) = 0 \]

So, \( x = 5 \) and \( x = -4 \)

are the non-permissible values.

c) \( \frac{x(2x - 3)}{4x^2 + 17x - 15} \)

\[ 4x^2 + 17x - 15 = 0 \]

\[ 4x^2 + 20x - 3x - 15 = 0 \]

\[ 4x(x + 5) - 3(x + 5) = 0 \]

\[ (4x - 3)(x + 5) = 0 \]

So, \( x = 5 \) and \( x = \frac{3}{4} \)

are the non-permissible values.
9. Which of these rational expressions are defined for all real values of \( x \)? Explain how you know.

a) \( \frac{2x^2 + 3}{6x} \)

\( x = 0 \) is a non-permissible value, so the expression is not defined for all values of \( x \).

b) \( \frac{3x + 7}{x^2 - 9} \)

\( x = 3 \) and \( x = -3 \) are non-permissible values, so the expression is not defined for all values of \( x \).

c) \( \frac{3x^2 + 2x - 1}{x^2 + 49} \)

Since \( x^2 \geq 0 \), \( x^2 + 49 > 0 \); Since the denominator cannot equal 0, the expression is defined for all values of \( x \).

d) \( \frac{x^2 + 4}{x^2 + 1} \)

\( x = -1 \) is a non-permissible value, so the expression is not defined for all values of \( x \).

10. Use multiplication and division to write two equivalent forms of each rational expression.

a) \( \frac{2x}{2x + 4} \)

The expression has \( x = -2 \) as a non-permissible value.

\[
\frac{2x}{2x + 4} = \frac{2x}{(2x + 4)} \cdot \frac{(x + 1)}{(x + 1)} = \frac{2x(x + 1)}{(2x + 4)(x + 1)}
\]

This expression has an additional non-permissible value: \( x = -1 \)

\[
\frac{2x}{2x + 4} = \frac{2x}{x + 2}
\]

The equivalent expressions are:

\[
\frac{2x(x + 1)}{(2x + 4)(x + 1)}, x \neq -2, -1
\]

\[
\frac{x}{x + 2}, x \neq -2
\]

b) \( \frac{3(x + 5)}{x(x + 8)(x + 5)} \)

This expression has \( x = 0 \), \( x = -8 \), and \( x = -5 \) as non-permissible values.

\[
\frac{3(x + 5)}{x(x + 8)(x + 5)} = \frac{3(x + 5)}{x(x + 8)(x + 5)} \cdot \frac{(x + 5)}{(x + 5)} = \frac{3(x + 5)^2}{x(x + 8)(x + 5)^2}
\]

\[
\frac{3(x + 5)}{x(x + 8)(x + 5)} = \frac{3(x + 5)}{x(x + 8)} = \frac{3}{x(x + 8)}
\]

The equivalent expressions are:

\[
\frac{3(x + 5)}{x(x + 8)(x + 5)^2}, x \neq -8, -5, 0
\]

and \( \frac{3}{x(x + 8)}, x \neq -8, -5, 0 \)
11. a) Write each rational expression in simplest form.

i) \( \frac{-p^1q^2}{5p^2q^3} \)

The non-permissible values are: \( p = 0 \) and \( q = 0 \)

\[ \frac{-p^1q^2}{5p^2q^3} = \frac{-p}{5} \cdot p \neq 0, q \neq 0 \]

ii) \( \frac{4x - 9}{x^2 - 9} \)

The non-permissible values are:
\( x = -3 \) and \( x = 3 \)

The numerator and denominator have no common factors, so the expression is in simplest form:
\( \frac{4x - 9}{x^2 - 9} \cdot x \neq -3, 3 \)

iii) \( \frac{2x^2 + 4x^2}{6x^2 - 24} \)

\[ \frac{2x^2(x + 2)}{6(x^2 - 4)} = \frac{2x^2(x + 2)}{6(x - 2)(x + 2)} \]

The non-permissible values are:
\( x = -2 \) and \( x = 2 \)

\[ \frac{2x^2(x + 2)}{3(x - 2)} \cdot x \neq -2, 2 \]

iv) \( \frac{36 - 9x^2}{x^2 - 5x + 6} \)

\[ \frac{9(4 - x^2)}{(x - 2)(x + 3)} = \frac{9(4 - x^2)}{x - 2)(x - 3)} \]

The non-permissible values are:
\( x = 2 \) and \( x = 3 \)

\[ \frac{9(4 - x^2)}{x - 3, x \neq 2, 3} \]

b) Choose one expression from part a. Explain why the non-permissible values of the given expression and its simplest form are the same.

In part iii, the numerator and denominator of the expression were divided by the common factor \( x + 2 \). This division is not possible when \( x = -2 \). So, \( x = -2 \) must be included in the non-permissible values of the simplified expression.

12. Here is a student’s solution for simplifying a rational expression. Identify the error in the solution. Write a correct solution.

\[ \frac{3x - 12}{x^2 + x - 20} = \frac{3(x - 4)}{(x + 5)(x - 4)} \]

\[ = \frac{3}{x + 5} \cdot x \neq -5 \]

The error in the solution is that \( x = 4 \) should be included as a non-permissible value. Division by \( x - 4 \) is not possible when \( x = 4 \).

Correct solution:

\[ \frac{3x - 12}{x^2 + x - 20} = \frac{3(x - 4)}{(x + 5)(x - 4)} \]

\[ = \frac{3}{(x + 5)} \cdot x \neq -5, 4 \]
13. A student claims that the expressions $\frac{12x^2}{15x}$ and $\frac{12x(x - 3)}{15(x - 3)}$ are equivalent. Is the student correct? Explain.

No, the student is not correct. When $x = 3$, the first expression equals 2.4 and the second expression is not defined. So, the expressions are not equivalent. The non-permissible values of both expressions must be stated.

14. Create a rational expression that has each set of non-permissible values. Explain how you created each expression.

a) $x \neq 0, x \neq 6$

Choose a denominator so that when $x = 0$ or 6, the value of the denominator is 0, and the value of the denominator is non-zero for all other values of $x$; for example, $x(x - 6)$

Then write a polynomial in the numerator: $\frac{x^2 + 1}{x(x - 6)}$

b) $x \neq 4, x \neq -7$

Choose a denominator so that when $x = 4$ or $-7$, the value of the denominator is 0, and the value of the denominator is non-zero for all other values of $x$; for example, $(x - 4)(x + 7)$

Then write a polynomial in the numerator: $\frac{-2x^2 + x}{(x - 4)(x + 7)}$

15. Write each rational expression in simplest form.

State the non-permissible values of the variables.

a) $\frac{2x^2 - 7xy + 6y^2}{x^3 - 16y^3}$

\[
= \frac{2x^2 - 4xy - 3xy + 6y^2}{(x^2 - 4y^2)(x^2 + 4y^2)}
= \frac{2x(x - 2y) - 3y(x - 2y)}{(x - 2y)(x + 2y)(x^2 + 4y^2)}
= \frac{(2x - 3y)(x - 2y)}{(x - 2y)(x + 2y)(x^2 + 4y^2)}
\]

The non-permissible values are:

$x = 2y$ and $x = -2y$

\[
= \frac{(2x - 3y)(x - 2y)}{(x - 2y)(x + 2y)(x^2 + 4y^2)}
= \frac{2x - 3y}{(x + 2y)(x^2 + 4y^2)}
\]

$x \neq 2y, -2y$

b) $\frac{x^4 - 5x^2 + 4}{x^3 + 3x^2 + 2x}$

\[
= \frac{(x^4 - 4)(x^2 - 1)}{x(x^2 + 3x + 2)}
= \frac{(x - 2)(x + 2)(x - 1)(x + 1)}{x(x + 1)(x + 2)}
\]

The non-permissible values are:

$x = 0, x = -2, and x = -1$

\[
= \frac{(x - 2)(x + 2)(x - 1)(x + 1)}{x(x + 1)(x + 2)}
= \frac{(x - 2)(x - 1)}{x}, x \neq -2, -1, 0
\]