Lesson 6.3 Math Lab: Assess Your Understanding, pages 468-469

1. In \( \triangle ABC \), \( AB = 4 \text{ cm} \) and \( \angle A = 70^\circ \)

   a) Sketch a diagram to show that there are two triangles with
   \( BC = 3.8 \text{ cm} \).

   b) To the nearest degree, measure \( \angle C \) for each triangle.
   \( \angle C = 98^\circ \) or \( 82^\circ \)

   c) To the nearest hundredth of a centimetre, calculate the length of
   \( BC \) for which \( \triangle ABC \) is a right triangle.

   If \( \triangle ABC \) is a right triangle, then
   \[
   \sin A = \frac{BC}{AB} \\
   \sin 70^\circ = \frac{BC}{4} \\
   BC = 4 \sin 70^\circ \\
   BC = 3.7587\ldots
   \]
   To the nearest hundredth of a centimetre, \( BC = 3.76 \text{ cm} \)

2. In \( \triangle ABC \), \( AB = 4 \text{ cm} \), \( BC = 3.5 \text{ cm} \), and \( \angle A = 70^\circ \).
   Use the completed chart in Part D to justify that it is not possible to draw a triangle.

   \[
   \frac{BC}{AB} = \frac{3.5}{4}, \text{ or } 0.875 \\
   \sin 70^\circ = 0.940 \\
   \text{Since } \frac{BC}{AB} < \sin 70^\circ, \text{ then no triangle is possible}
3. In ΔABC, AB = 4 cm and ∠A = 70°
   a) Choose a value for BC for which a unique triangle that is not a right triangle can be drawn. Draw the triangle.

   **Sample response:**
   \[
   \begin{align*}
   \text{Sample response: I chose BC = 5 cm.} \\
   \frac{BC}{AB} = \frac{5}{4} = 1.25 \\
   \text{Since } \frac{BC}{AB} > 1, \text{ then only one triangle is possible.}
   \end{align*}
   \]

   b) Use the completed chart in Part D to justify that only one scalene triangle can be drawn with the value you chose for BC.

   **Sample response:** I chose BC = 5 cm.

4. In ΔABC, AB = 10 cm and BC = 8 cm; to the nearest degree, determine possible measures of acute ∠A for each situation.
   a) No triangle is possible.

   \[
   \begin{align*}
   &\angle A \text{ is acute, so } \angle A < 90° \\
   &\text{For no triangle, } \sin A > \frac{8}{10}, \text{ or } 0.8 \\
   &\sin^{-1}(0.8) = 53° \\
   &\text{For an acute angle } \theta, \text{ as } \theta \text{ increases, } \sin \theta \text{ also increases.} \\
   &\text{So, for no triangle, } 53° < \angle A < 90°
   \end{align*}
   \]

   b) One right triangle is possible.

   \[
   \begin{align*}
   &\text{For a right triangle, } \frac{BC}{AB} = \sin A \\
   &\sin A = 0.8 \\
   &\text{From part a, } \angle A \approx 53°; \text{ so, for one right triangle, } \angle A = 53°
   \end{align*}
   \]

   c) Two scalene triangles are possible.

   \[
   \begin{align*}
   &\text{For two scalene triangles, } \sin A < \frac{BC}{AB} < 1 \\
   &\frac{BC}{AB} = 0.8, \text{ so } \sin A < 0.8 \\
   &\text{From part a, } \angle A < 53°; \text{ so, for two scalene triangles, } \angle A < 53°
   \end{align*}
   \]