Lesson 5.5 Exercises, pages 393–398

4. Simplify each expression. Use a calculator to verify the answer.

a) \( \log 6 + \log 5 \)
   
   \[
   = \log (6 \cdot 5) \\
   = \log 30 \\
   \text{Verify: } \log 6 + \log 5 = 1.4771\ldots \\
   \log 30 = 1.4771\ldots
   \]

b) \( 3 \log 2 \)
   
   \[
   = \log 2^3 \\
   = \log 8 \\
   \text{Verify: } 3 \log 2 = 0.9030\ldots \\
   \log 8 = 0.9030\ldots
   \]

c) \( \log 48 - \log 6 \)
   
   \[
   = \log \left( \frac{48}{6} \right) \\
   = \log 8 \\
   \text{Verify: } \log 48 - \log 6 = 0.9030\ldots \\
   \log 8 = 0.9030\ldots
   \]

d) \( \log 8 + \log 5 \)
   
   \[
   = \log (8 \cdot 5) \\
   = \log 40 \\
   \text{Verify: } \log 8 + \log 5 = 1.6020\ldots \\
   \log 40 = 1.6020\ldots
   \]
5. Write each expression as a single logarithm.

a) \( \log a - \log b \)
   \[ = \log \left( \frac{a}{b} \right) \]

b) \( \log x + \log y \)
   \[ = \log xy \]

c) \( 5 \log m \)
   \[ = \log m^5 \]

d) \( \log x - \log y + \log z \)
   \[ = \log \left( \frac{x}{y} \right) + \log z \]
   \[ = \log \left( \frac{xz}{y} \right) \]

6. Use each law of logarithms to write an expression that is equal to \( \log 16 \). Use a calculator to verify each expression.

a) the product law
   Sample response: Determine 2 numbers whose product is 16: 2 and 8
   So, \( \log 16 = \log 2 + \log 8 \)
   Verify: \( \log 16 = 1.2041\ldots \) \( \log 2 + \log 8 = 1.2041\ldots \)

b) the quotient law
   Determine 2 numbers whose quotient is 16: 80 and 5
   So, \( \log 16 = \log 80 - \log 5 \)
   Verify: \( \log 16 = 1.2041\ldots \) \( \log 80 - \log 5 = 1.2041\ldots \)

c) the power law
   Write 16 as a power: \( 4^2 \)
   So, \( \log 16 = 2 \log 4 \)
   Verify: \( \log 16 = 1.2041\ldots \) \( 2 \log 4 = 1.2041\ldots \)

7. Substitute values of \( a \) and \( b \) to verify each statement.

a) \( \frac{\log a}{\log b} \neq \log \left( \frac{a}{b} \right) \)
   Substitute: \( a = 3 \) and \( b = 5 \)
   \( \log a = \log 3 \)
   \( \log b = \log 5 \)
   \[ = 0.6826\ldots \]
   \( \log \left( \frac{a}{b} \right) = \log \left( \frac{3}{5} \right) \)
   \[ = -0.2218\ldots \]
   Since the left side is not equal to the right side, the statement is verified.

b) \( \log (a + b) \neq \log ab \)
   Substitute: \( a = 3 \) and \( b = 5 \)
   \( \log (a + b) = \log 8 \)
   \[ = 0.9030\ldots \]
   \( \log ab = \log 15 \)
   \[ = 1.1760\ldots \]
   Since the left side is not equal to the right side, the statement is verified.
8. Write each expression as a single logarithm.
   a) \( \log x - 5 \log y \) 
   \[ = \log x - \log y^5 \]
   \[ = \log \left( \frac{x}{y^5} \right) \]

   b) \( \frac{1}{2} \log x + 3 \log y \)
   \[ = \log x^{1/2} + \log y^3 \]
   \[ = \log \left( x^{1/2}y^3 \right) \]

   c) \( \frac{2}{3} \log x - 4 \log y - 3 \log z \)
   \[ = \log x^{2/3} - \log y^4 - \log z^3 \]
   \[ = \log \left( \frac{x^{2/3}}{y^4z^3} \right) \]

   d) \( 5 + \log c \)
   \[ = \log 2^5 + \log c \]
   \[ = \log 32 + \log c \]
   \[ = \log_{32} c \]

9. Explain each step in this proof of the power law for logarithms.
   To prove that \( \log_{b} x^k = k \log_{b} x \):

   Let: \( \log_{b} x = n \)  Write the logarithm as a power.

   Then: \( x = b^n \)  Raise each side to the power \( k \).

   \[ x^k = (b^n)^k \]  Simplify.

   \[ x^k = b^{kn} \]  Take the logarithm base \( b \) of each side.

   \[ \log_{b} x^k = \log_{b} b^{kn} \]  Simplify the right side.

   \[ \log_{b} x^k = kn \]  Substitute: \( n = \log_{b} x \)

   \[ \log_{b} x^k = k \log_{b} x \]

10. Use the strategy from the proof of the product law for logarithms to prove the quotient law: \( \log_{b} \left( \frac{x}{y} \right) = \log_{b} x - \log_{b} y \)

    Let \( \log_{b} x = m \) and \( \log_{b} y = n \)  Apply the definition of a logarithm.

    Then \( x = b^m \) and \( y = b^n \)  Use the quotient law for exponents.

    So \( \frac{x}{y} = \frac{b^m}{b^n} \)

    \[ \frac{x}{y} = b^{m-n} \]  Write this exponential statement as a logarithmic statement.

    \[ \log_{b} \left( \frac{x}{y} \right) = m - n \]  Substitute for \( m \) and \( n \).

    \[ \log_{b} \left( \frac{x}{y} \right) = \log_{b} x - \log_{b} y \]
11. Use two different strategies to write $2\left(\log x + \log y\right)$ as a single logarithm.

**Strategy 1**

\[
2\left(\log x + \log y\right) = 2 \log x + 2 \log y = \log x^2 + \log y^2 = \log x^2y^2
\]

**Strategy 2**

\[
2\left(\log x + \log y\right) = 2 \log xy = \log (xy)^2
\]

12. Write each expression as a single logarithm.

a) $3 \log 2 + \log 6$

\[
= \log 2^3 + \log 6 = \log (8 \cdot 6) = \log 48
\]

c) $3 \log 6 - 2$

\[
= \log 6^3 - \log 2^2 = \log 216 - \log 4 = \log \left(\frac{216}{4}\right) = \log 54
\]

d) $5 \log 2 - \log 4 + 2$

\[
= \log 2^5 - \log 4 + \log 4 = \log 32 - \log 4 + \log 25 = \log \frac{32 \cdot 25}{4} = \log 200
\]

13. Evaluate each expression.

a) $2 \log 6 - 3 \log 2 + \log 18$

\[
= \log 6^2 - \log 2^3 + \log 18 = \log \left(\frac{36 \cdot 18}{8}\right) = \log 81 = \log 3^4 = 4
\]

c) $9 \log 3 - \log 75 + 2 \log 5$

\[
= \log 3^9 - \log 75 + \log 5^2 = \log \left(\frac{19683 \cdot 25}{75}\right) = \log 6561 = \log 9^4 = 4
\]

d) $\log 98 - 2 \log 7 - 2$

\[
= \log 98 - \log 7^2 - 2 = \log 98 - \log 49 - 2 = \log \left(\frac{98}{49}\right) = \log 2 = \frac{1}{2} - 2 = -3 - \frac{1}{2}
\]
14. Given \( \log a = 1.301 \), determine an approximate value for each logarithm.

a) \( \log a^3 \)  
\[ = 3 \log a \]  
\[ = 3(1.301) \]  
\[ = 3.903 \]

b) \( \log 10a \)  
\[ = \log 10 + \log a \]  
\[ = 1 + 1.301 \]  
\[ = 2.301 \]

c) \( \log \left( \frac{a^2}{100} \right) \)  
\[ = \log a^2 - \log 100 \]  
\[ = 2 \log a - 2 \]  
\[ \approx 2(1.301) - 2 \]  
\[ \approx 0.602 \]

15. Identify the errors in the solution to the question below. Write the correct solution.

Write \( \log \left( \frac{a^2}{c^b} \right) \) in terms of \( \log a \), \( \log b \), and \( \log c \).

\[
\log \left( \frac{a^2}{c^b} \right) = \log a^2 - \log c^b = 2 \log a - (\log c + \log b) \\
= \frac{1}{2} \log a - \log 3c + 2 \log b = \log a^2 - \log c - \log b \\
= \frac{1}{2} \log a - 3 \log c - 2 \log b
\]

In the second line of the solution, when \( \log c^b \) is written as a sum of logarithms, both logarithms should be negative. In the third line of the solution, \( \log c^2 \) should be \( 3 \log c \).

16. Write each expression in terms of \( \log a \), \( \log b \), and/or \( \log c \).

a) \( \log a^3 b^2 \)
\[
= \log a^3 + \log b^2 \\
= 3 \log a + \frac{1}{2} \log b
\]

b) \( \log ab^2 c^2 \)
\[
= \log a + \log b^2 + \log c^2 \\
= \log a + 2 \log b + \frac{3}{2} \log c
\]

c) \( \log \left( \frac{a^2}{b^3} \right) \)
\[
= \log a^2 - \log b^3 \\
= 3 \log a - 2 \log b
\]

d) \( \log \left( \frac{a^4 b^3}{c} \right) \)
\[
= \log a^4 + \log b^3 - \log c \\
= 4 \log a + \frac{3}{2} \log b - \log c
\]
17. Given \( \log 3 = 0.477 \) and \( \log 7 = 0.845 \), determine the approximate value of \( \log (132300) \) without using a calculator.

Write 132300 as a product of factors that involve powers of 3 and 7.

\[
132300 \div 9 = 14700
\]
\[
14700 \div 3 = 4900
\]
\[
4900 \div 49 = 100
\]

So, \( 132300 \approx 3^3 \cdot 7^2 \cdot 10^2 \)

\[
\log (132300) = \log (3^3 \cdot 7^2 \cdot 10^2)
\]
\[
= \log 3^3 + \log 7^2 + \log 10^2
\]
\[
= 3 \log 3 + 2 \log 7 + 2
\]
\[
\approx 3(0.477) + 2(0.845) + 2
\]
\[
\approx 5.121
\]

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18. Write each expression as a single logarithm.

a) \( 3 \log x + \log (2x - 3) \)

\[
= \log x^3 + \log (2x - 3)
\]
\[
= \log x^3(2x - 3)
\]

b) \( \log (x + 1) + \log (2x - 1) \)

\[
= \log (x + 1)(2x - 1)
\]

19. Without using the power law, prove the law of logarithms for radicals:

\[
\log_b \sqrt[k]{x} = \frac{1}{k} \log_b x, \ b > 0, b \neq 1, k \in \mathbb{N}, x > 0
\]

Sample response:

Let: \( \log_b x = n \)

Write the logarithm as a power.

Then \( x = b^n \)

Raise each side to the power \( \frac{1}{k} \).

\[
x^\frac{1}{k} = b^n \cdot \frac{1}{k}
\]

Take the logarithm base \( b \) of each side.

\[
\log_b x^\frac{1}{k} = \log_b b^n \cdot \frac{1}{k}
\]

Simplify the right side.

\[
\log_b x^\frac{1}{k} = \frac{n}{k}
\]

Substitute: \( n = \log_b x \)

\[
\log_b x^\frac{1}{k} = \frac{1}{k} \log_b x
\]

Or, \( \log_b \sqrt[k]{x} = \frac{1}{k} \log_b x \)