Lesson 5.3 Exercises, pages 364–368

A

3. Write each number as a power of 2.
   a) 16   b) 128   c) \( \frac{1}{32} \)   d) 1
   \[
   = 2^4 \quad = 2^7 \quad = 2^{-5} \quad = 2^0
   \]

4. Which numbers below can be written as powers of 5? Write each number you identify as a power of 5.
   a) 125   b) 10   c) \( \frac{1}{25} \)   d) 1
   \[
   = 5^3 \quad \text{cannot be written as a power of 5} \quad = 5^{-2} \quad = 5^0
   \]

5. Write each number as a power of 3.
   a) \( \sqrt[3]{9} \)   b) \( \sqrt[3]{243} \)   c) \( \frac{\sqrt[3]{9}}{3} \)   d) \( 27\sqrt[3]{3} \)
   \[
   = 9^{\frac{1}{3}} \quad = 243^{\frac{1}{3}} \quad = 3^{\frac{1}{3}} \cdot 3^{\frac{1}{3}} \quad = 3^{3} \cdot 3^{\frac{1}{3}}
   \]
   \[
   = (3^3)^{\frac{1}{3}} \quad = (3^3)^{\frac{1}{3}} \quad = 3^{1+1} \quad = 3^{3+1}
   \]
   \[
   = 3^{\frac{1}{3}} \quad = 3^{\frac{1}{2}} \quad = 3^{\frac{1}{2}} \quad = 3^{\frac{1}{2}}
   \]

B

6. Solve each equation.
   a) \( 2^x = 256 \)
   \[
   2^8 \quad x = 8
   \]
   b) \( 81 = 3^{x+1} \)
   \[
   3^4 = 3^{x+1} \quad 4 = x + 1 \quad x = 3
   \]
   c) \( 3^x = 9^{x-2} \)
   \[
   3^x = (3^2)^{x-2} \quad x = 2(x - 2) \quad x = 2x - 4 \quad x = 4
   \]
   d) \( 4^{x-1} = 2^{x+3} \)
   \[
   (2^2)^{x-1} = 2^{x+3} \quad 2^x - 1 = x + 3 \quad 2x - 1 = x + 3 \quad x = 5
   \]
   e) \( 8^{2x} = 16^{x+3} \)
   \[
   (2^3)^{2x} = (2^4)^{x+3} \quad 3(2x) = 4(x + 3) \quad 6x = 4x + 12 \quad 2x = 12 \quad x = 6
   \]
   f) \( 9^{x+1} = 243^{x+3} \)
   \[
   (3^2)^{x+1} = (3^5)^{x+3} \quad 2(x + 1) = 5(x + 3) \quad 2x + 2 = 5x + 15 \quad -3x = 13 \quad x = \frac{13}{3}
   \]
7. Use graphing technology to solve each equation. Give the solution to the nearest tenth.

a) \(10 = 2^x\)
   Graph: \(y = 2^x - 10\)
   The approximate zero is \(x = 3.3\)

b) \(3^x = 100\)
   Graph: \(y = 100 - 3^x\)
   The approximate zero is \(x \approx 4.2\)

c) \(3^{x+1} = 50\)
   Graph: \(y = 50 - 3^{x+1}\)
   The approximate zero is \(x \approx 2.6\)

d) \(30 = 2^{x-1}\)
   Graph: \(y = 2^{x-1} - 30\)
   The approximate zero is \(x \approx 5.9\)

8. Explain why the equation \(4^x = -2\) does not have a real solution. Verify, graphically, that there is no solution.

The value of a power with a positive base can never be negative, so the equation does not have a real solution. When I graph \(y = -2 - 4^x\), the graph does not have an \(x\)-intercept.

9. Solve each equation.

a) \(2^x = 8\sqrt{2}\)
   \[2^x = 2^1 \cdot 2^{\frac{3}{2}}\]
   \[2^x = 2^{1 + \frac{3}{2}}\]
   \[x = 3 + \frac{1}{3}\]
   \[x = \frac{10}{3}\]

b) \(81\sqrt{3} = 3^x\)
   \[3^4 \cdot 3^{\frac{1}{2}} = 3^x\]
   \[3^4 + \frac{1}{2} = 3^x\]
   \[4 + \frac{1}{2} = x\]
   \[x = \frac{9}{2}\]

c) \(2^{x+1} = 2\sqrt{4}\)
   \[2^{x+1} = 2 \cdot 2^{\frac{1}{2}}\]
   \[2^{x+1} = 2(2^x)^{\frac{1}{2}}\]
   \[2^x = 2^{1 - 2}\]
   \[2^x = 2^{1 + \frac{1}{2}}\]
   \[x + 1 = 1 + \frac{2}{3}\]
   \[x = \frac{2}{3}\]

d) \(9^x = \sqrt{27}\)
   \[(3^2)^x = (3^3)^{\frac{1}{2}}\]
   \[2x = \frac{3}{2}\]
   \[x = \frac{3}{4}\]

e) \(\sqrt{216} = 36^{x-1}\)
   \[216^{\frac{1}{2}} = 6^{2(x-1)}\]
   \[\left(\frac{3}{4}\right)^2 = 2x - 2\]
   \[2x = \frac{11}{4}\]
   \[x = \frac{11}{8}\]

f) \((\sqrt{7})^{x+1} = \sqrt{49}\)
   \[7^{\frac{1}{2}(x+1)} = (7^3)^{\frac{1}{2}}\]
   \[\frac{1}{2}x + \frac{1}{2} = \frac{2}{3}\]
   \[3x + 3 = 4\]
   \[3x = 1\]
   \[x = \frac{1}{3}\]
10. Solve each equation.

a) \(\left(\frac{1}{2}\right)^x = 2^4\)
\[
2^{-4x} = 2^4 \\
(-2)(3) = x \\
x = -6
\]

b) \(5^x = \frac{\sqrt{25}}{25}\)
\[
5^x = (5^{\frac{1}{2}})^2 \cdot 5^{-2} \\
x = \frac{2}{3} - 2 \\
x = -\frac{4}{3}
\]

c) \(\frac{\sqrt[4]{49}}{3\sqrt[3]{13}} = 7^{x+1}\)
\[
\left(7^{\frac{1}{2}}\right) \cdot 7^{-1} = 7^{x+1} \\
7^{\frac{1}{2} - 1} = 7^{x+1} \\
\frac{2}{3} - 3 = x + 1 \\
-\frac{7}{3} = x + 1 \\
x = -\frac{10}{3}
\]

d) \(\left(\frac{1}{9}\right)^x = 3\sqrt[3]{7}\)
\[
(3^{-3})^x = 3^1 \cdot (3^3)^{\frac{1}{3}} \\
3^{-2x} = 3^{1+\frac{1}{3}} \\
-2x = 1 + \frac{1}{3} \\
-2x = \frac{4}{3} \\
x = -\frac{2}{3}
\]

e) \(8^{1-x} = \frac{\sqrt[4]{16}}{4}\)
\[
2^{3(1-x)} = (2^3)^{\frac{1}{4}} \cdot 2^{-2} \\
2^{3(1-x)} = 2^{1-2} \\
3 - 3x = \frac{4}{3} - 2 \\
-3x = -\frac{11}{3} \\
x = \frac{11}{9}
\]

f) \(\left(\frac{1}{8}\right)^{x+1} = \left(\sqrt[3]{16}\right)^x\)
\[
(2^{-3})^{x+1} = (2^3)^{\frac{1}{3}} \\
-3x - 3 = \frac{4}{3}x \\
-3x = \frac{13}{3}x = 3 \\
x = 9 \frac{9}{13}
\]

11. Use graphing technology to solve each equation. Give the solution to the nearest tenth.

a) \(2 = 1.05^x\)
Graph: \(y = 1.05^x - 2\) 
The approximate zero is \(x = 14.2\)

b) \(2^{\frac{4}{5}} = 0.4\)
Graph: \(y = 0.4 - 2^{\frac{-4}{5}}\) 
The approximate zero is \(x = 6.6\)

(c) \(2^{x+1} = 3^{x-2}\)
Graph: \(y = 3^{x-2} - 2^{x+1}\) 
The approximate zero is \(x = 7.1\)

d) \(3(2^x) = 64\)
Graph: \(y = 64 - 3(2^x)\) 
The approximate zero is \(x = 4.4\)
12. A principal of $600 was invested in a term deposit that pays 5.5% annual interest, compounded semi-annually. To the nearest tenth of a year, when will the amount be $1000?

Use: \[ A = A_0 \left(1 + \frac{i}{n}\right)^{nt} \]
Substitute: \[ A = 1000, A_0 = 600, i = 0.055, n = 2 \]
\[ 1000 = 600 \left(1 + \frac{0.055}{2}\right)^{2t} \]
Graph \[ y = 600 \left(1 + \frac{0.055}{2}\right)^{2t} - 1000 \], then determine the zero of the function.

The approximate zero is: 9.4148676
It will take approximately 9.4 years for the term deposit to amount to $1000.

13. a) To the nearest year, how long will it take an investment of $500 to double at each annual interest rate, compounded annually?

i) 4%  
ii) 6%  
iii) 8%  
iv) 9%  
v) 12%

Use: \[ A = A_0 \left(1 + \frac{i}{n}\right)^{nt} \]
Substitute: \[ A = 1000, A_0 = 500, i = 0.055, n = 1 \]
\[ 1000 = 500 \left(1 + \frac{0.055}{1}\right)^t \]

Use this expression below.

i) Substitute: \[ i = 0.04 \]
\[ 2 = (1 + 0.04)^t \]
\[ 2 = 1.04^t \]
Graph \[ y = 1.04^t - 2 \], then determine the zero of the function.

The approximate zero is: 17.672988
It will take approximately 18 years.

ii) Substitute: \[ i = 0.06 \]
\[ 2 = (1 + 0.06)^t \]
\[ 2 = 1.06^t \]
Graph \[ y = 1.06^t - 2 \], then determine the zero of the function.

The approximate zero is: 11.895661
It will take approximately 12 years.

iii) Substitute: \[ i = 0.08 \]
\[ 2 = (1 + 0.08)^t \]
\[ 2 = 1.08^t \]
Graph \[ y = 1.08^t - 2 \], then determine the zero of the function.

The approximate zero is: 9.0064683
It will take approximately 9 years.

iv) Substitute: \[ i = 0.09 \]
\[ 2 = (1 + 0.09)^t \]
\[ 2 = 1.09^t \]
Graph \[ y = 1.09^t - 2 \], then determine the zero of the function.

The approximate zero is: 8.0432317
It will take approximately 8 years.

v) Substitute: \[ i = 0.12 \]
\[ 2 = (1 + 0.12)^t \]
\[ 2 = 1.12^t \]
Graph \[ y = 1.12^t - 2 \], then determine the zero of the function.

The approximate zero is: 6.1162554
It will take approximately 6 years.

b) What pattern is there in the interest rates and times in part a?

The product of each interest rate as a percent and time in years is 72.
14. When light passes through glass, the intensity is reduced by 5%.
   a) Determine a function that models the percent of light, \( P \), that passes through \( n \) layers of glass.

   For 0 layers of glass, the percent of light is: \( P = 100 \)
   For 1 layer of glass, the percent of light is: \( P = 100(0.95) \)
   For 2 layers of glass, the percent of light is: \( P = 100(0.95)^2 \)
   For 3 layers of glass, the percent of light is: \( P = 100(0.95)^3 \)
   For \( n \) layers of glass, the percent of light is: \( P = 100(0.95)^n \)

   b) Determine how many layers of glass are needed for only 25% of light to pass through.

   Solve the equation: \( 25 = 100(0.95)^n \)
   Graph a related function: \( y = 100(0.95)^x - 25 \)
   The approximate zero of the function is: 27.026815
   So, 27 layers of glass are needed.

C

15. Solve each equation, then verify the solution graphically.
   a) \( 2^{(x^2)} = 16 \)
   \[\begin{align*}
   x^2 &= 4 \\
   x &= \pm 2
   \end{align*}\]
   The graph of \( y = 16 - 2^{(x^2)} \) has \( x \)-intercepts 2 and -2.

   b) \( 9^{x+4} = 3^{(x^2)} \)
   \[\begin{align*}
   x^2 &= 4 \\
   2x + 8 &= x^2 \\
   x^2 - 2x - 8 &= 0 \\
   (x - 4)(x + 2) &= 0 \\
   x &= 4 \text{ or } x = -2
   \end{align*}\]
   A graph of \( y = 3^{(x^2)} - 9^{x+4} \) has \( x \)-intercepts -2 and 4.

16. For what values of \( k \) does the equation \( 9^{(x^2)} = 27^{x+k} \) have no real solution?

   \[\begin{align*}
   9^{(x^2)} &= 27^{x+k} \\
   3^{(2x^2)} &= 3^{(3x+k)} \\
   2x^2 &= 3x + 3k \\
   2x^2 - 3x - 3k &= 0
   \end{align*}\]
   For no real roots, the discriminant is less than 0.
   \[\begin{align*}
   (-3)^2 - 4(2)(-3k) &< 0 \\
   9 &< -24k \\
   k &< -\frac{9}{24} \text{ or } -\frac{3}{8}
   \end{align*}\]
   The equation has no real solution when \( k < -\frac{3}{8} \).