Checkpoint: Assess Your Understanding, pages 213–218

3.1

1. **Multiple Choice** The graph of \( y = -3x^3 + 4 \) is translated 4 units right and 5 units down. What is an equation of the translation image?

   A. \( y = -3(x + 4)^3 + 9 \)  
   B. \( y = -3(x - 4)^3 + 9 \)  
   C. \( y = -3(x + 4)^3 - 1 \)  
   D. \( y = -3(x - 4)^3 - 1 \)
2. Here is the graph of \( y = g(x) \). On the same grid, sketch the graph of each function below. State the domain and range of each function.

a) \( y - 3 = g(x) \)

Compare the equation to 
\[ y - k = g(x): k = 3 \]
So, mark some lattice points on \( y = g(x) \) and translate each point 3 units up.
Both functions have domain: \( x \in \mathbb{R} \)
Both functions have range: \( y \in \mathbb{R} \)

b) \( y = g(x + 2) \)

Write \( y = g(x + 2) \) as \( y = g(x - (-2)) \).
Compare the equation to \( y = g(x - h): h = -2 \)
Translate each point on the graph of \( y = g(x) \) 2 units left.
The domain is: \( x \in \mathbb{R} \)
The range is: \( y \in \mathbb{R} \)

c) \( y + 1 = g(x - 3) \)

Write \( y + 1 = g(x - 3) \) as \( y - (-1) = g(x - 3) \).
Compare the equation to \( y - k = g(x - h): h = 3 \) and \( k = -1 \)
Translate each point on the graph of \( y = g(x) \) 3 units right and 1 unit down.
The domain is: \( x \in \mathbb{R} \)
The range is: \( y \in \mathbb{R} \)
3. The graph of \( y = f(x) \) was translated to create each graph below. Write an equation of each graph in terms of the function \( f \).

a) The graph of \( y = f(x) \) has a local maximum at (2, 9).
This graph has a local maximum at \((-3, 6)\).
So, the graph of \( y = f(x) \) was translated 5 units left and
3 units down.
The equation of the image graph has the form:
\[ y - k = f(x - h), \text{ where} \]
\( h = -5 \) and \( k = -3 \)
So, an equation of the image graph is: \( y + 3 = f(x + 5) \)

b) The graph of \( y = f(x) \) has a local maximum at (2, 9).
This graph has a local maximum at (5, 11).
So, the graph of \( y = f(x) \) was translated 3 units right and
2 units up.
The equation of the image graph has the form: \( y - k = f(x - h), \text{ where} \]
\( h = 3 \) and \( k = 2 \)
So, an equation of the image graph is: \( y - 2 = f(x - 3) \)
3.2

4. **Multiple Choice** The graph of \( y = f(x) \) was reflected in the \( x \)-axis. Which graph below is its reflection image?

![Graphs A, B, C, D]

A. \[ y \]

B. \[ y \]

C. \[ y \]

D. \[ y \]

5. Here is the graph of \( y = k(x) \). On the same grid, sketch and label the graph of each function below, then state its domain and range.

a) \( y = -k(x) \)

The graph of \( y = -k(x) \) is the image of the graph of \( y = k(x) \) after a reflection in the \( x \)-axis.

Mark some lattice points on \( y = k(x) \), then reflect them in the \( x \)-axis. Mark these image points, then join them.

Domain: \( x \in \mathbb{R} \)

Range: \( y \leq -2 \)

b) \( y = k(-x) \)

The graph of \( y = k(-x) \) is the image of the graph of \( y = k(x) \) after a reflection in the \( y \)-axis.

Mark some lattice points on \( y = k(x) \), then reflect them in the \( y \)-axis. Mark these image points, then join them.

Domain: \( x \in \mathbb{R} \)

Range: \( y \geq 2 \)
6. The graph of \( y = -x^3 + 3x^2 - x + 3 \) was reflected in the \( y \)-axis and its image is shown. What is an equation of the image?

When the graph of \( y = f(x) \) is reflected in the \( y \)-axis, the equation of its image is \( y = f(-x) \).

So, an equation of the image is:
\[
\begin{align*}
y &= f(-x) \\
y &= -(-x)^3 + 3(-x)^2 - (-x) + 3 \\
y &= x^3 + 3x^2 + x + 3
\end{align*}
\]

3.3

7. Multiple Choice The point \((-6, 2)\) lies on the graph of \( y = f(x) \). After vertical and horizontal stretches or compressions of the graph, the equation of the image is \( y = 3f(2x) \). Which point is the image of \((-6, 2)\)?

A. \((-3, 6)\)  
B. \((-12, 6)\)  
C. \((-2, 4)\)  
D. \((-18, 1)\)

8. Here is the graph of \( y = h(x) \). On the same grid, sketch the graph of each function below, then state its domain and range.

a) \( y = \frac{1}{3}h(-2x) \)

Compare \( y = ah(bx) \) to \( y = \frac{1}{3}h(-2x) \): \( a = \frac{1}{3} \) and \( b = -2 \)

So, the graph of \( y = h(x) \) is vertically compressed by a factor of \( \frac{1}{3} \), horizontally compressed by a factor of \( \frac{1}{2} \), then reflected in the \( y \)-axis. Use mental math and the transformation: \((x, y)\) on \( y = h(x) \) corresponds to \( (\frac{x}{2}, \frac{1}{3}y) \) on \( y = \frac{1}{3}h(-2x) \), to mark some image points, then join them.

Domain: \(-4 \leq x \leq 2\); range: \(-1 \leq y \leq 2\)

b) \( y = 2h(4x) \)

Compare \( y = ah(bx) \) to \( y = 2h(4x) \): \( a = 2 \) and \( b = 4 \)

So, the graph of \( y = h(x) \) is vertically stretched by a factor of \( 2 \), and horizontally compressed by a factor of \( \frac{1}{4} \). Use mental math and the transformation: \((x, y)\) on \( y = h(x) \) corresponds to \( (\frac{x}{4}, 2y) \) on \( y = 2h(4x) \), to mark some image points, then join them.

Domain: \(-1 \leq x \leq 2\); range: \(-6 \leq y \leq 12\)
9. The graph of \( y = g(x) \) is the image of the graph of \( y = f(x) \) after a vertical and/or horizontal stretch and/or reflection. Corresponding points are labelled. Write an equation of the image graph in terms of the function \( f \).

Point \( A(4, 2) \) on \( y = f(x) \) corresponds to point \( A'(8, -6) \) on \( y = g(x) \).

An equation for the image graph after a vertical or horizontal stretch or compression can be written in the form \( y = af(bx) \).

A point \((x, y)\) on \( y = f(x) \) corresponds to the point \(\left(\frac{x}{b}, ay\right)\) on \( y = af(bx) \).

The image of \( A(4, 2) \) is \(\left(\frac{4}{b}, a(2)\right)\), which is \( A'(8, -6) \).

Equate the \( x \)-coordinates: \( b = \frac{1}{2} \)

Equate the \( y \)-coordinates: \( a = -3 \)

So, an equation of \( y = g(x) \) is: \( y = -3f\left(\frac{1}{2}x\right) \)

I used the coordinates of \( B \) and \( B' \), and mental math to verify the equation.