Lesson 3.1 Exercises, pages 169–175

A

4. On each grid, the graph of \( y = |x| \) and its image after a single translation are shown. What was the translation? What is the equation of the image graph?

a) The graph of \( y = |x| \) was translated 2 units right. The image graph has equation \( y = |x - 2| \).

b) The graph of \( y = |x| \) was translated 4 units down. The image graph has equation \( y + 4 = |x| \).

5. For each equation of a translation image, describe how the graph of \( y = f(x) \) was translated.

a) \( y = f(x - 7) \)
   - Compare the equation to \( y = f(x - h) \): \( h = 7 \)
   - So, the graph of \( y = f(x) \) was translated 7 units right.

b) \( y + 5 = f(x) \)
   - Write \( y + 5 = f(x) \)
   - as \( y - (-5) = f(x) \).
   - Compare the equation to \( y - k = f(x) \): \( k = -5 \)
   - So, the graph of \( y = f(x) \) was translated 5 units down.
c) \( y = f(x + 6) \)

Write \( y = f(x + 6) \) as \( y = f(x - (-6)) \).

Compare the equation to \( y = f(x - h); h = -6 \).

So, the graph of \( y = f(x) \) was translated 6 units left.

6. The graph of \( y = g(x) \) is translated as described below. Write the equation of each translation image in terms of the function \( g \).

a) a translation of 3 units right

The translation is horizontal, so the equation of the image graph has the form \( y = g(x - h) \). The translation is 3 units right, so \( h = 3 \). The equation of the translation image is: \( y = g(x - 3) \)

b) a translation of 8 units down

The translation is vertical, so the equation of the image graph has the form \( y - k = g(x) \). The translation is 8 units down, so \( k = -8 \). The equation of the translation image is: \( y - (-8) = g(x) \), or \( y + 8 = g(x) \)

c) a translation of 9 units up

The translation is vertical, so the equation of the image graph has the form \( y - k = g(x) \). The translation is 9 units up, so \( k = 9 \). The equation of the translation image is: \( y - 9 = g(x) \)

d) a translation of 7 units left

The translation is horizontal, so the equation of the image graph has the form \( y = g(x - h) \). The translation is 7 units left, so \( h = -7 \). The equation of the translation image is: \( y = g(x - (-7)) \), or \( y = g(x + 7) \)

7. Here is the graph of \( y = g(x) \). On the same grid, sketch the graph of each function below. State the domain and range of each function.

a) \( y - 1 = g(x) \)

Compare the equation to \( y - k = g(x); k = 1 \)

So, translate each point on the graph of \( y = g(x) \) 1 unit up.

Both functions have the same domain: \(-2 \leq x \leq 5\)

The range of \( y = g(x) \) is: \(-1 \leq y \leq 3\)

The range of \( y - 1 = g(x) \) is: \(0 \leq y \leq 4\)

b) \( y = g(x + 3) \)

Write \( y = g(x + 3) \) as \( y = g(x - (-3)) \).

Compare the equation to \( y = g(x - h); h = -3 \)

So, translate each point on the graph of \( y = g(x) \) 3 units left.

The domain of \( y = g(x) \) is: \(-2 \leq x \leq 5\)

The domain of \( y = g(x + 3) \) is: \(-5 \leq x \leq 2\)

Both functions have the same range: \(-1 \leq y \leq 3\)
8. Here is the graph of \( y = f(x) \). On the same grid, sketch the graph of each function below. State the domain and range of each function.

![Graph](image)

a) \( y + 3 = f(x + 2) \)

Write \( y + 3 = f(x + 2) \) as \( y - (-3) = f(x - (-2)) \).

Compare the equation to \( y - k = f(x - h) \):

\[ h = -2 \text{ and } k = -3 \]

Mark some lattice points on \( y = f(x) \).

Translate each point 2 units left and 3 units down, then join the points with a smooth curve.

Both functions have domain: \( x \in \mathbb{R} \)

The range of \( y = f(x) \) is: \( y \geq 0 \)

The range of \( y + 3 = f(x + 2) \) is: \( y \geq -3 \)

b) \( y - 1 = f(x - 2) \)

Compare the equation to \( y - k = f(x - h) \):

\[ h = 2 \text{ and } k = 1 \]

Mark some lattice points on \( y = f(x) \).

Translate each point 2 units right and 1 unit up, then join the points with a smooth curve.

Both functions have domain: \( x \in \mathbb{R} \)

The range of \( y = f(x) \) is: \( y \geq 0 \)

The range of \( y - 1 = f(x - 2) \) is: \( y \geq 1 \)

9. The point \( A(3, 7) \) lies on the graph of \( y = f(x) \). What are the coordinates of its image \( A' \) on the graph of \( y - 2 = f(x - 8) \)? How do you know?

Compare the equation \( y - 2 = f(x - 8) \) to \( y - k = f(x - h) \):

\[ h = 8 \text{ and } k = 2 \]

So, each point on the graph of \( y = f(x) \) is translated 8 units right and 2 units up to create the graph of \( y - 2 = f(x - 8) \).

So, the image of point \( A \) is \( A'(3 + 8, 7 + 2) \), or \( A'(11, 9) \).
10. Here is the graph of $y = j(x)$. On the same grid, sketch the graph of $y - 3 = j(x + 2)$. Describe how the vertical and horizontal asymptotes are affected by the translations. What are the equations of the asymptotes of the image graph?

$$\text{Compare } y - 3 = j(x + 2) \text{ to } y - k = j(x - h):$$

$$h = -2 \text{ and } k = 3$$

So, translate the vertical asymptotes 2 units left and the horizontal asymptote 3 units up.

Then sketch the graph of $y - 3 = j(x + 2)$ so that it is congruent to the graph of $y = j(x)$.

The equations of the vertical asymptotes are $x = -4$ and $x = 0$.

The equation of the horizontal asymptote is $y = 3$.

11. Use the graph of $y = p(x)$ to sketch the graph of each function below.

a) $y + 2 = p(x + 3)$

$$\text{Compare } y + 2 = p(x + 3) \text{ to } y - k = p(x - h):$$

$$h = -3 \text{ and } k = -2$$

Mark some lattice points on $y = p(x)$. Translate each point 3 units left and 2 units down. Draw a smooth curve through the points so that the image graph is congruent to the graph of $y = p(x)$.

b) $y - 3 = p(x - 2)$

$$\text{Compare } y - 3 = p(x - 2) \text{ to } y - k = p(x - h):$$

$$h = 2 \text{ and } k = 3$$

Mark some lattice points on $y = p(x)$. Translate each point 2 units right and 3 units up. Draw a smooth curve through the points so that the image graph is congruent to the graph of $y = p(x)$.
12. The function \( y = f(x) \) has domain \(-7 \leq x \leq 12\) and range \(-1 \leq y \leq 10\). What are the domain and range of \( y + 8 = f(x - 3)\)?

Write \( y + 8 = f(x - 3) \) as \( y - (-8) = f(x - 3)\).

Compare the equation to \( y - k = f(x - h) \); \( h = 3 \) and \( k = -8 \)

So, each point on the graph of \( y = f(x) \) is translated 3 units right and 8 units down to create the graph of \( y + 8 = f(x - 3)\).

So, the image graph has domain: \(-7 + 3 = -4 \leq x \leq 12 + 3 = 15\), or \(-4 \leq x \leq 15\)

and range: \(-1 - 8 = -9 \leq y \leq 10 - 8 = 2\), or \(-9 \leq y \leq 2\)

13. Describe how the graph of \( y = \sqrt{x} \) has been translated to create the graph of each function below. Use graphing technology to check.

a) \( y = \sqrt{x - 1} \)

Compare \( y = \sqrt{x - 1} \) to \( y - k = \sqrt{x - h} \); \( h = 1 \) and \( k = 0 \)

So, the graph of \( y = \sqrt{x - 1} \) is the graph of \( y = \sqrt{x} \) after a translation of 1 unit right.

b) \( y = \sqrt{x + 4} - 2 \)

Write \( y = \sqrt{x + 4} - 2 \) as \( y - (-2) = \sqrt{x - (-4)} \), then compare to

\( y - k = \sqrt{x - h} \); \( h = -4 \) and \( k = -2 \)

So, the graph of \( y = \sqrt{x + 4} - 2 \) is the graph of \( y = \sqrt{x} \) after a
translation of 4 units left and 2 units down.

14. The graph of \( y = x^3 \) is translated 5 units left and 3 units up. What is the equation of the image graph?

The equation of the image graph has the form \( y - k = (x - h)^3 \).

\( h = -5 \) and \( k = 3 \)

So, the equation of the image graph is: \( y - 3 = (x - (-5))^3 \), or \( y - 3 = (x + 5)^3 \)

15. The graph of \( y = |x^2 - 2| \) is translated 2 units right and 7 units down. What is the equation of the image graph?

The equation of the image graph has the form \( y - k = |(x - h)^2 - 2| \).

\( h = 2 \) and \( k = -7 \)

So, the equation of the image graph is: \( y - (-7) = |(x - 2)^2 - 2| \), or \( y + 7 = |(x - 2)^2 - 2| \)
16. The graph of \( y = f(x) \) has been translated to create the graphs below. Match each graph to its equation.

\[
\begin{align*}
\text{i)} & \quad \text{Graph A} \\
\text{ii)} & \quad \text{Graph B} \\
\text{iii)} & \quad \text{Graph C} \\
\text{iv)} & \quad \text{Graph D}
\end{align*}
\]

\[
\begin{align*}
a) & \quad y - 1 = f(x + 2) \\
& \quad \text{Compare } y - 1 = f(x - (2)) \text{ to } y - k = f(x - h); h = -2 \text{ and } k = 1 \\
& \quad \text{So, the graph of } y = f(x) \text{ is translated 2 units left and 1 unit up.} \\
& \quad \text{This corresponds to Graph D.}
\end{align*}
\]

\[
\begin{align*}
b) & \quad y - 1 = f(x - 2) \\
& \quad \text{Compare } y - 1 = f(x - 2) \text{ to } y - k = f(x - h); h = 2 \text{ and } k = 1 \\
& \quad \text{So, the graph of } y = f(x) \text{ is translated 2 units right and 1 unit up.} \\
& \quad \text{This corresponds to Graph A.}
\end{align*}
\]

\[
\begin{align*}
c) & \quad y + 1 = f(x - 2) \\
& \quad \text{Compare } y - (1) = f(x - 2) \text{ to } y - k = f(x - h); h = 2 \text{ and } k = -1 \\
& \quad \text{So, the graph of } y = f(x) \text{ is translated 2 units right and 1 unit down.} \\
& \quad \text{This corresponds to Graph B.}
\end{align*}
\]

\[
\begin{align*}
d) & \quad y + 2 = f(x - 1) \\
& \quad \text{Compare } y + 2 = f(x - 1) \text{ to } y - k = f(x - h); h = 1 \text{ and } k = -2 \\
& \quad \text{So, the graph of } y = f(x) \text{ is translated 1 unit right and 2 units down.} \\
& \quad \text{This corresponds to Graph C.}
\end{align*}
\]
17. The graph of $f(x) = x^2 - 5x - 6$ has zeros at $-1$ and $6$.
   a) The graph of $y = f(x)$ is translated horizontally. Neither zero of the image graph is negative. What is the shortest possible translation?
      A translation of 1 unit right would move the zero $-1$ to $0$. So, the shortest possible translation is 1 unit right.
   b) The graph of $y = f(x)$ is translated horizontally. Neither zero of the image graph is positive. What is the shortest possible translation?
      A translation of 6 units left would move the zero $6$ to $0$. So, the shortest possible translation is 6 units left.

C

18. The graph of $y = 2x^2 - 12x + 23$ is translated vertically so that its vertex lies on the x-axis. How could you use the discriminant to determine the translation? What is the translation?

The equation of the translation image has the form $y - k = 2x^2 - 12x + 23$, or $y = 2x^2 - 12x + 23 + k$.
The vertex lies on the x-axis when the value of the discriminant is 0.
In $b^2 - 4ac$, substitute: $a = 2$, $b = -12$, $c = 23 + k$
\((-12)^2 - 4(2)(23 + k) = 0\)
\(144 - 8(23 + k) = 0\)
\(-8(23 + k) = -144\)
\(23 + k = 18\)
\(k = -5\)
So, the translation is 5 units down.