1. Use long division to divide. Write the division statement.

\[ (-x^2 - 5x^3 + 2x - 3x^4 + 3) \div (x - 2) \]

Write the polynomial in descending order:
\[ -3x^4 - 5x^3 - x^2 + 2x + 3 \]

\[ x - 2 ] \underline{\quad -3x^4 - 5x^3 - x^2 + 2x + 3} \]
\[ -3x^4 - 11x^3 - 23x - 44 \]
\[ -11x^3 - x^2 \]
\[ -11x^3 + 22x^2 \]
\[ -23x^2 + 2x \]
\[ -23x^2 + 46x \]
\[ -44x + 3 \]
\[ -44x + 88 \]
\[ -85 \]

\[ -3x^4 - 5x^3 - x^2 + 2x + 3 = (x - 2)(-3x^3 - 11x^2 - 23x - 44) - 85 \]

2. Use synthetic division to divide. Write the division statement.

\[ (7x^4 + x^3 - 8x^2) \div (x - 5) \]

Write the polynomial in descending order:
\[ x^4 - 8x^3 + 7x \]

Use zeros as placeholders.

\[ \begin{array}{c|cccc} 5 & 1 & -8 & 0 & 7 & 0 \\ \hline & 5 & -15 & -75 & -340 & \end{array} \]

\[ 1 -3 & -15 & -68 & -340 \]

\[ x^4 - 8x^3 + 7x = (x - 5)(x^3 - 3x^2 - 15x - 68) - 340 \]

3. Determine the remainder: \( (4x^4 - 7x^3 + 2x^2 + x) \div (x + 1) \)

\[ x + 1 = x - (-1) \]

Let \( P(x) = 4x^4 - 7x^3 + 2x^2 + x \)

\[ P(-1) = 4(-1)^4 - 7(-1)^3 + 2(-1)^2 + (-1) \]
\[ = 4 + 7 + 2 - 1 \]
\[ = 12 \]

The remainder is 12.
4. Fully factor this polynomial: \(6x^3 - 11x^2 - 3x + 2\)

Let \(P(x) = 6x^3 - 11x^2 - 3x + 2\)

The factors of 2 are: 1, −1, 2, −2
Use mental math to substitute \(x = 1\), then \(x = -1\)
to determine that neither \(x - 1\) nor \(x + 1\) is a factor.

Try \(x = 2\): \(P(2) = 6(2)^3 - 11(2)^2 - 3(2) + 2 = 0\)

So, \(x - 2\) is a factor.

Divide to determine the other factor.

\[
\begin{array}{cccc}
2 & 6 & -11 & -3 & 2 \\
& 12 & 2 & -2 \\
6 & 1 & -1 & 0 \\
\end{array}
\]

So, \(6x^3 - 11x^2 - 3x + 2 = (x - 2)(6x^2 + x - 1)\)

Factor the trinomial: \(6x^2 + x - 1 = (3x - 1)(2x + 1)\)

So, \(6x^3 - 11x^2 - 3x + 2 = (x - 2)(3x - 1)(2x + 1)\)

5. Sketch a graph of each polynomial function.

a) \(f(x) = x^4 - x^3 - 4x^2 + 4x\)

The equation represents an even-degree polynomial function.
The leading coefficient is positive, so the graph opens up.
The constant term is 0, so the \(y\)-intercept is 0.
Make a table of values. Plot the points then join them with a smooth curve.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(f(x))</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>60</td>
</tr>
<tr>
<td>-2</td>
<td>0</td>
</tr>
<tr>
<td>-1</td>
<td>-6</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>30</td>
</tr>
</tbody>
</table>
b) \( g(x) = (x + 2)^3(x - 2)^2 \)

To determine the roots, let \( g(x) = 0 \).

0 = \((x + 2)^3(x - 2)^2\)

Zeros of the function are: -2 and 2

The zero -2 has multiplicity 3.

The zero 2 has multiplicity 2.

So, the graph crosses the x-axis at \( x = -2 \) and just touches the x-axis at \( x = 2 \).

The equation has degree 5, so it is an odd-degree polynomial function.

The leading coefficient is positive, so as \( x \to -\infty \), the graph falls and as \( x \to \infty \), the graph rises.

The y-intercept is: \( (2)^3(-2)^2 = 32 \)

Plot points at the intercepts then join them with a smooth curve.

2

6. For the graph of the quadratic function \( y = f(x) \) below:

a) Sketch the graph of \( y = \sqrt{f(x)} \).

Mark points where \( y = 0 \) or \( y = 1 \).

Choose, then mark other points on the graph of \( y = \sqrt{f(x)} \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y = f(x) )</th>
<th>( y = \sqrt{f(x)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>6</td>
<td>( \sqrt{6} \approx 2.4 )</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>( \sqrt{6} \approx 2.4 )</td>
</tr>
</tbody>
</table>

Join all the points with 2 smooth curves.

b) State the domain and range of \( y = \sqrt{f(x)} \).

Domain is: \( x \leq -1, x \geq 3 \)

Range is: \( y \geq 0 \)

7. Use graphing technology to solve each equation. Give the solution to the nearest tenth.

a) \( \sqrt{4 - x} + 3 = 2 + \sqrt{2x} \)

Graph the related function:

\( f(x) = \sqrt{4 - x} + 1 - \sqrt{2x} \)

The approximate zero is: 2.4867591

So, the root is: \( x = 2.5 \)

b) \( \frac{-3}{x^2 - 2} = \frac{5}{x - 2} \)

Graph the related function:

\( f(x) = \frac{-3}{x^2 - 2} - \frac{5}{x - 2} \)

The approximate zeros are:

\(-2.113836 \) and \( 1.5138357 \)

So, the roots are: \( x \approx -2.1 \) and \( x \approx 1.5 \)
8. Match each function below to its graph. Justify your choice.

i) Graph A

\[ y = \frac{x^2 - 2x - 3}{x + 3} \]

The numerator can be factored.
\[ y = \frac{(x - 3)(x + 1)}{x + 3} \]
There are no common factors. Since \( x + 3 \neq 0, x = -3 \) is a vertical asymptote. The degree of the numerator is 1 more than that of the denominator, so there is an oblique asymptote. The function matches Graph B.

ii) Graph B

\[ y = \frac{x^2 - 2}{x^2 - 4} \]

Since \( x^2 - 4 \neq 0, x \neq \pm 2, \) so \( x = 2 \) and \( x = -2 \) are vertical asymptotes. The degrees of the numerator and denominator are the same so there is a horizontal asymptote. As \( |x| \to \infty, \)
\[ y = \frac{x^2 - 2}{x^2 - 4} \to y = \frac{x^2}{x^2} \text{ or } y = 1, \]
which is the horizontal asymptote. The function matches Graph A.

iii) Graph C

\[ y = \frac{x^2 - 2x - 3}{x - 3} \]

The numerator can be factored.
\[ y = \frac{(x - 3)(x + 1)}{x - 3} \]
Since the numerator and denominator have a common factor, there is a hole where the denominator \( x - 3 = 0; \) that is, at \( x = 3. \) The function matches Graph C.