1. **Multiple Choice** Given the graph of the function $y = f(x)$, which graph below right represents $y = \sqrt{f(x)}$?
2. For each function \( y = f(x) \) graphed below:
- Sketch the graph of \( y = \sqrt{f(x)} \).
- State the domain and range of \( y = \sqrt{f(x)} \).
- Explain why the domains are different and the ranges are different.

![Graphs of functions](image)

Mark points where \( y = 0 \) or \( y = 1 \). The graph of \( y = \sqrt{f(x)} \) is above the graph of \( y = f(x) \) between these points. Choose, then mark other points on the graph of \( y = \sqrt{f(x)} \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y = f(x) )</th>
<th>( y = \sqrt{f(x)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>2</td>
<td>( \approx 1.4 )</td>
</tr>
<tr>
<td>0</td>
<td>4</td>
<td>2</td>
</tr>
</tbody>
</table>

Join the points with a smooth curve.
Domain is: \( x \geq -6 \)
Range is: \( y \geq 0 \)

\( y = \sqrt{f(x)} \) is not defined for \(-2 < x < 6\). Mark points where \( y = 0 \) or \( y = 1 \). Choose, then mark other points on the graph of \( y = \sqrt{f(x)} \).

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<tr>
<td>-4</td>
<td>5</td>
<td>( \approx 2.2 )</td>
</tr>
<tr>
<td>8</td>
<td>5</td>
<td>( \approx 2.2 )</td>
</tr>
</tbody>
</table>

Join the points with 2 smooth curves.
Domain is: \( x \leq -2 \) or \( x \geq 6 \)
Range is: \( y \geq 0 \)

The domain of a linear function or a quadratic function is all real values of \( x \), but the square root of a negative number is undefined, so any value of \( x \) that makes the radicand negative is not in the domain of a radical function.
The range of the linear function is all real values of \( y \), and the range of the quadratic function is all real values of \( y \) that are greater than or equal to \(-4\). The principal square root of a number is always 0 or positive, so the range of the radical functions is restricted to these values of \( y \).
3. Solve each radical equation by graphing. Give the solution to the nearest tenth.

a) \(-x + 3 = \sqrt{2x - 1}\)
   Write the equation as: 
   \(-x + 3 - \sqrt{2x - 1} = 0\)
   Graph the related function: 
   \(f(x) = -x + 3 - \sqrt{2x - 1}\)
   Use graphing technology to determine the approximate zero: 1.5505103
   So, the solution is: \(x \approx 1.6\)

b) \(\sqrt{x + 2} = 5 - \sqrt{3x + 4}\)
   Write the equation as: 
   \(\sqrt{x + 2} - 5 + \sqrt{3x + 4} = 0\)
   Graph the related function: 
   \(f(x) = \sqrt{x + 2} - 5 + \sqrt{3x + 4}\)
   Use graphing technology to determine the approximate zero: 1.779514
   So, the solution is: \(x \approx 1.8\)

2.2

4. Use graphing technology to graph each rational function.

Identify any non-permissible values of \(x\) and the equations of any horizontal asymptotes.

a) \(y = \frac{3x}{x + 4}\)
   Since \(x + 4 \neq 0\), then \(x \neq -4\)
   The vertical asymptote has equation \(x = -4\).
   The horizontal asymptote has equation \(y = 3\).

b) \(y = \frac{3x}{x^2 - 4}\)
   Since \(x^2 - 4 \neq 0\), then \(x \neq \pm 2\)
   The vertical asymptotes have equations \(x = 2\) and \(x = -2\).
   The horizontal asymptote has equation \(y = 0\).

c) \(y = \frac{x^2 - 4}{3x}\)
   Since \(3x \neq 0\), then \(x \neq 0\)
   The vertical asymptote has equation \(x = 0\).
   There is no horizontal asymptote.

d) \(y = \frac{x^2 - 4x}{3x}\)
   Since \(3x \neq 0\), then \(x \neq 0\)
   There is a hole at \(x = 0\).
   There is no horizontal asymptote.

2.3

5. Multiple Choice Which function has a graph with a hole?

A. \(y = \frac{x + 4}{2x^2 + 8x}\)  
B. \(y = \frac{x - 4}{2x^2 + 8x}\)

C. \(y = \frac{4x + 4}{2x^2 + 8x}\)  
D. \(y = \frac{x + 4}{2x^2 - 8x}\)
6. Match each function to its graph. Justify your choice.

i) Graph A

\[ y = \frac{-12x}{x - 3} \]

There is a vertical asymptote with equation \( x = 3 \). The degrees of the numerator and denominator are equal, so there is a horizontal asymptote that is not the \( x \)-axis. The function matches Graph B.

ii) Graph B

\[ y = \frac{2x^2 - x - 15}{3 - x} \]

Factor: \[ y = \frac{(2x + 5)(x - 3)}{3 - x} \]

or \[ y = -\frac{(2x + 5)(x - 3)}{x - 3} \]

There is a hole at \( x = 3 \). The function matches Graph C.

iii) Graph C

\[ y = \frac{x^2}{x - 3} \]

There is a vertical asymptote with equation \( x = 3 \). The degree of the numerator is 1 more than the degree of the denominator, so there is an oblique asymptote. The function matches Graph D.

iv) Graph D

\[ y = \frac{x^2}{x^2 - 3} \]

The function is not defined for \( x^2 - 3 = 0 \); that is, \( x = \pm \sqrt{3} \). So, there are vertical asymptotes at \( x = -\sqrt{3} \) and \( x = \sqrt{3} \). The degrees of the numerator and denominator are equal, so there is a horizontal asymptote that is not the \( x \)-axis. The function matches Graph A.
7. For the graph of each rational function below, determine without technology:
   i) the equations of any asymptotes and the coordinates of any hole
   ii) the domain of the function
   Use graphing technology to verify the characteristics.

   a) \( y = \frac{2x^2}{25 - x^2} \)
      i) The function is undefined when \( 25 - x^2 = 0 \); that is, when \( x = \pm 5 \). There are no common factors, so there are vertical asymptotes with equations \( x = 5 \) and \( x = -5 \).
      The degrees of the numerator and denominator are equal, so there is a horizontal asymptote. The leading coefficients of the numerator and denominator are 2 and \(-1\), respectively.
      So, the horizontal asymptote has equation: \( y = -2 \)
      ii) The domain is: \( x \neq \pm 5 \)

   b) \( y = \frac{-2x^2 - 6x}{x + 3} \)
      i) The function is undefined when \( x + 3 = 0 \); that is, when \( x = -3 \).
      Factor: \( y = \frac{-2x(x + 3)}{x + 3} \)
      There is a hole at \( x = -3 \). The function is: \( y = -2x, x \neq -3 \)
      The coordinates of the hole are: \((-3, 6)\)
      ii) The domain is: \( x \neq -3 \)

8. Solve each rational equation by graphing. Give the solution to the nearest tenth.

   a) \( x - 2 = \frac{3x - 5}{x - 3} \)
      Graph a related function: \( f(x) = x - 2 - \frac{3x - 5}{x - 3} \)
      Use graphing technology to determine the zeros:
      \( x = 1.8 \) or \( x = 6.2 \)

   b) \( \frac{x^2 + 3x - 5}{x - 1} = -5 \)
      Graph a related function: \( f(x) = x^2 + 3x - 5 + 5 \)
      Use graphing technology to determine the zeros:
      \( x = -9.1 \) or \( x = 1.1 \)