1. **Multiple Choice** Which statement is true?
   A. When \(2x^3 + 4x^2 - 2x - 1\) is divided by \(x - 2\), the remainder is 3.
   B. The binomial \(x + 1\) is a factor of \(4x^4 - x^3 - 3x + 2\).
   C. When \(2x^4 - 7x^3 + 6x^2 - 14x + 20\) is divided by \(x - 3\), the remainder is \(-5\).
   D. The binomial \(x + 2\) is a factor of \(5x^3 + 7x^2 + 12\).

2. **Multiple Choice** Which statement about the graph of a quartic function is false?
   A. The graph may open up.
   B. The graph may have a zero of multiplicity 3.
   C. The graph may fall to the left and rise to the right.
   D. The graph may have a zero of multiplicity 2.

3. Divide \(2x^4 + 11x^3 - 10 - 5x + 14x^2\) by \(x + 2\).
   Write the division statement.
   \[
   \begin{array}{c|cccc}
   & -2 & 2 & 11 & 14 & -5 & -10 \\
   \hline
   2 & 7 & 0 & -5 & 0 \\
   \end{array}
   \]
   \[2x^4 + 11x^3 + 14x^2 - 5x - 10 = (x + 2)(2x^3 + 7x^2 - 5)\]

4. Does the polynomial \(x^4 - x^3 - 14x^2 + x + 16\) have a factor of \(x + 3\)? How do you know?
   Let \(P(x) = x^4 - x^3 - 14x^2 + x + 16\)
   \[P(-3) = (-3)^4 - (-3)^3 - 14(-3)^2 - 3 + 16\]
   \[= 81 + 27 - 126 - 3 + 16\]
   \[= -5\]
   \(P(-3) \neq 0\), so \(x + 3\) is not a factor of the polynomial.

5. Factor: \(4x^4 - 20x^3 + 17x^2 + 26x - 15\)
   Let \(P(x) = 4x^4 - 20x^3 + 17x^2 + 26x - 15\)
   The factors of \(-15\) are: 1, -1, 3, -3, 5, -5, 15, -15
   Use mental math to determine that \(x - 1\) is not a factor and \(x + 1\) is a factor. Divide to determine the other factor.
6. Sketch the graph of this polynomial function.

\( g(x) = 4x^4 + 11x^3 - 7x^2 - 11x + 3 \)

**Factor the polynomial.**

The factors of the constant term, 3, are: 1, -1, 3, -3

Use mental math to substitute \( x = 1 \) in \( g(x) \) to determine that both \( x - 1 \) and \( x + 1 \) are factors.

Divide by \( x - 1 \).

| -1 | 4 | -20 | 17 | 26 | -15 \\
|----|---|-----|----|----|------
| 4  | -4 | 24  | -41| 15  |

So, \( 4x^4 - 20x^3 + 17x^2 + 26x - 15 = (x + 1)(4x^3 - 24x^2 + 41x - 15) \)

Let \( P(x) = 4x^3 - 24x^2 + 41x - 15 \)

Try \( x = 3 \): \( P(3) = 4(3)^3 - 24(3)^2 + 41(3) - 15 = 0 \)

So, \( x - 3 \) is a factor. Divide to determine the other factor.

<table>
<thead>
<tr>
<th>3</th>
<th>4</th>
<th>-24</th>
<th>41</th>
<th>-15</th>
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<tr>
<td>4</td>
<td>-12</td>
<td>5</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

Factor the trinomial: \( 4x^2 - 12x + 5 = (2x - 5)(2x - 1) \)

So, \( 4x^4 - 20x^3 + 17x^2 + 26x - 15 = (x + 1)(x - 3)(2x - 5)(2x - 1) \)

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**Factor the trinomial:**

\( 4x^2 + 11x - 3 = (x - 1)(4x^2 + 15x^2 + 8x - 3) \)

Divide \( 4x^2 + 15x^2 + 8x - 3 \) by \( x + 1 \).

<table>
<thead>
<tr>
<th>-1</th>
<th>4</th>
<th>15</th>
<th>8</th>
<th>-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>-4</td>
<td>11</td>
<td>-3</td>
<td>0</td>
</tr>
</tbody>
</table>

So, \( 4x^2 + 11x - 3 = (x - 1)(4x + 1)(4x^2 + 11x - 3) \)

Factor the trinomial:

\( 4x^2 + 11x - 3 = (x + 3)(4x - 1) \)

So, \( 4x^6 + 11x^5 - 7x^4 - 11x + 3 \)

= \( (x - 1)(x + 1)(4x + 1)(4x^2 + 11x - 3) \)

The zeros of \( g(x) \) are:

1, -1, -3, 0.25

So, the \( x \)-intercepts of the graph are:

1, -1, -3, 0.25

Each zero has multiplicity 1, so the graph crosses the \( x \)-axis at each \( x \)-intercept. The equation has degree 4, so it is an even-degree polynomial function. The leading coefficient is positive, so the graph opens up. The constant term is 3, so the \( y \)-intercept is 3.
7. Canada Post defines a small packet as one for which the sum of its length, width, and height is less than or equal to 90 cm. A company produces several different small packets, each with length 15 cm longer than its height.

a) Write a polynomial function to represent possible volumes of one of these packets in terms of its height \( x \). Assume the sum of the dimensions is maximized.

    The sum of the length, width, and height of the packet is 90 cm.
    The formula for the volume, \( V \), of the packet is: \( V = lwh \)
    Let \( x \) represent the height of the packet, in centimetres.
    Then the length, in centimetres, is \( x + 15 \), and the width, in centimetres, is \( 90 - (x + x + 15) \), or \( 75 - 2x \).
    So, \( V(x) = x(x + 15)(75 - 2x) \)

b) Graph the function.

    Enter the equation: \( y = x(x + 15)(75 - 2x) \) into a graphing calculator.

c) To the nearest cubic centimetre, what is the maximum possible volume of one of these packets? What are its dimensions to the nearest tenth of a centimetre?

    Determine the coordinates of the local maximum point:
    \( (23.1124\ldots, 25.347.18\ldots) \)
    The maximum volume of the packet is approximately 25 347 cm\(^3\).
    This occurs when the height of the packet is approximately 23.1 cm.
    So, the length is approximately 23.1 cm + 15 cm = 38.1 cm and the width is: 75 cm – 2(23.1124\ldots cm) = 28.8 cm.
    The maximum volume of one of these packets is approximately 25 347 cm\(^3\). Its dimensions are approximately 38.1 cm by 28.8 cm by 23.1 cm.