1.1

1. **Multiple Choice** When synthetic division is used to divide \(x^4 - 7x^2 + 2x + 3\) by \(x + 1\), the result is: \(1 \ -1 \ -6 \ 8 \ -5\)

Which is the correct division statement?

A. \(x^4 - 7x^2 + 2x + 3 = (x + 1)(x^3 - x^2 - 6x + 8) + 0\)

B. \(x^4 - 7x^2 + 2x + 3 = (x + 1)(x^3 - x^2 - 6x + 8) - 5\)

C. \(x^4 - 7x^2 + 2x + 3 = (x + 1)(x^3 - x^2 - 6x + 8) + 5\)

D. \(x^4 - 7x^2 + 2x + 2 = (x + 1)(x^3 - x^2 - 6x + 8)\)

2. a) Use long division to divide:

i) \(2x^3 - 11x - 19\) by \(x - 7\)

\[
\begin{array}{c|cccc}
   & 2x^2 + 3 & -3x - 19 & \\
\hline
   x - 7 | 2x^3 & -11x & -19 \\
   & 2x^2 & -14x & \\
   & 3x & -19 & \\
   & 3x & -21 & 2
\end{array}
\]

The quotient is \(2x + 3\) and the remainder is 2.

ii) \(3x^3 + 4x^2 - 15x + 30\) by \(x + 4\)

\[
\begin{array}{c|cccc}
   & 3x^2 - 8x + 17 & -8x^2 - 32x & \\
\hline
   x + 4 | 3x^3 & +4x^2 & -15x & +30 \\
   & 3x^2 & +12x & \\
   & -8x^2 & -15x & \\
   & -8x^2 & -32x & 17x + 30 \\
   & 17x & +68 & \\
   & 17x & +68 & -38
\end{array}
\]

The quotient is \(3x^2 - 8x + 17\) and the remainder is \(-38\).
b) Verify the answer to one division in part a.

Sample response:
In part ii, multiply the quotient by the divisor, then add the remainder.

\[(x + 4)(3x^2 - 8x + 17) + (-38)\]
\[= 3x^3 - 8x^2 + 17x + 12x^2 - 32x + 68 - 38\]
\[= 3x^3 + 4x^2 - 15x + 30\]
Since this is the original polynomial, the answer is correct.

3. Use synthetic division to divide \(2x^3 - 7x^2 - 29x - 8 + 12\) by each binomial.

a) \(x - 1\)

\[
\begin{array}{c|ccccc}
1 & 2 & -7 & -29 & -8 & 12 \\
\hline
 & 1 & -5 & -34 & -42 & \\
\end{array}
\]

The quotient is \(2x^2 - 5x - 34\) and the remainder is \(-30\).

b) \(x - 6\)

\[
\begin{array}{c|ccccc}
6 & 2 & -7 & -29 & -8 & 12 \\
\hline
 & 12 & 30 & 6 & -12 & \\
\end{array}
\]

The quotient is \(2x^2 + 5x^0 + x - 2\) and the remainder is 0.

c) \(x + 2\)

\[
\begin{array}{c|ccccc}
-2 & 2 & -7 & -29 & -8 & 12 \\
\hline
 & -4 & 22 & 14 & -12 & \\
\end{array}
\]

The quotient is \(2x^2 - 11x^0 - 7x + 6\) and the remainder is 0.

d) \(x + 3\)

\[
\begin{array}{c|ccccc}
-3 & 2 & -7 & -29 & -8 & 12 \\
\hline
 & -6 & 39 & -30 & 114 & \\
\end{array}
\]

The quotient is \(2x^2 - 13x^0 + 10x - 38\) and the remainder is 126.
1.2

4. **Multiple Choice** When a polynomial $P(x)$ is divided by $x + 3$, the remainder is $-4$. Which statement is true?

A. $P(-4) = -3$  
B. $P(3) = -4$  
C. $P(-3) = -4$  
D. $P(-3) = 0$

5. **a)** Determine the remainder when $3x^4 + 8x^3 - 15x^2 - 32x + 12$ is divided by $x + 1$.

Let $P(x) = 3x^4 + 8x^3 - 15x^2 - 32x + 12$

$P(-1) = 3(-1)^4 + 8(-1)^3 - 15(-1)^2 - 32(-1) + 12$

$= 3 - 8 - 15 + 32 + 12$

$= 24$

The remainder is 24.

**b)** Is $x + 1$ a factor of the polynomial in part a? If your answer is yes, explain how you know. If your answer is no, determine a binomial of the form $x - a$, $a \in \mathbb{Z}$, that is a factor.

$x + 1$ is not a factor of the polynomial because the remainder after dividing by $x + 1$ is not 0.

The factors of the constant term, 12, are: 1, $-1$, 2, $-2$, 3, $-3$, 4, $-4$, 6, $-6$, 12, $-12$. Use mental math to substitute $x = 1$ to determine that $x - 1$ is not a factor.

Try $x = 2$: $P(2) = 3(2)^4 + 8(2)^3 - 15(2)^2 - 32(2) + 12$

$= 48 + 64 - 60 - 64 + 12$

$= 0$

So, $x - 2$ is a factor of $3x^4 + 8x^3 - 15x^2 - 32x + 12$.

6. For each polynomial, determine one factor of the form $x - a$, $a \in \mathbb{Z}$.

a) $x^3 - 5x^2 - 17x + 21$

Sample response:

Let $P(x) = x^3 - 5x^2 - 17x + 21$

The factors of 21 are: 1, $-1$, 3, $-3$, 7, $-7$, 21, $-21$

Try $x = 1$: $P(1) = (1)^3 - 5(1)^2 - 17(1) + 21$

$= 0$

So, $x - 1$ is a factor of $x^3 - 5x^2 - 17x + 21$.

b) $4x^4 - 15x^3 - 32x^2 + 33x + 10$

Sample response:

Let $P(x) = 4x^4 - 15x^3 - 32x^2 + 33x + 10$

The factors of 10 are: 1, $-1$, 2, $-2$, 5, $-5$, 10, $-10$

Try $x = 1$: $P(1) = 4(1)^4 - 15(1)^3 - 32(1)^2 + 33(1) + 10$

$= 0$

So, $x - 1$ is a factor of $4x^4 - 15x^3 - 32x^2 + 33x + 10$. 

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7. Factor this polynomial.

\[4x^4 - 12x^3 + 3x^2 + 13x - 6\]

Let \(P(x) = 4x^4 - 12x^3 + 3x^2 + 13x - 6\)
The factors of \(-6\) are: 1, -1, 2, -2, 3, -3, 6, -6

Try \(x = 1\):

\[P(1) = 4(1)^4 - 12(1)^3 + 3(1)^2 + 13(1) - 6 = 2\]

So, \(x - 1\) is not a factor.

Try \(x = -1\):

\[P(-1) = 4(-1)^4 - 12(-1)^3 + 3(-1)^2 + 13(-1) - 6 = 0\]

So, \(x + 1\) is a factor.

Divide to determine the other factor.

\[
\begin{array}{cccc}
-1 & 4 & -12 & 3 & 13 & -6 \\
 & -4 & 16 & -19 & 6 \\
4 & -16 & 19 & -6 & 0 \\
\end{array}
\]

So, \(4x^4 - 12x^3 + 3x^2 + 13x - 6 = (x + 1)(4x^3 - 16x^2 + 19x - 6)\)

Let \(P_1(x) = 4x^3 - 16x^2 + 19x - 6\)

Try \(x = 2\):

\[P_1(2) = 4(2)^3 - 16(2)^2 + 19(2) - 6 = 0\]

So, \(x - 2\) is a factor.

Divide to determine the other factor.

\[
\begin{array}{cccc}
2 & 4 & -16 & 19 & -6 \\
 & 8 & -16 & 6 \\
4 & -8 & 3 & 0 \\
\end{array}
\]

So, \(4x^4 - 12x^3 + 3x^2 + 13x - 6 = (x + 1)(x - 2)(4x^2 - 8x + 3)\)

Factor the trinomial: \(4x^2 - 8x + 3 = (2x - 1)(2x - 3)\)

So, \(4x^4 - 12x^3 + 3x^2 + 13x - 6 = (x + 1)(x - 2)(2x - 1)(2x - 3)\)