Lesson 5.2 Exercises, pages 360–368

3. Determine whether each point is a solution of the given inequality.
   a) $3x - 2y \geq -16$ \hspace{1cm} A(3, 4)

   In the inequality, substitute: $x = -3$, $y = 4$
   L.S.: $3(-3) - 2(4) = -17$ \hspace{1cm} R.S. = -16
   Since the L.S. < R.S., the point is not a solution.

   b) $4x - y \leq 5$ \hspace{1cm} B(-1, 1)

   In the inequality, substitute: $x = -1$, $y = 1$
   L.S.: $4(-1) - 1 = -5$ \hspace{1cm} R.S. = 5
   Since the L.S. < R.S., the point is a solution.

   c) $3y > 2x - 7$ \hspace{1cm} C(-2, -5)

   In the inequality, substitute: $x = -2$, $y = -5$
   L.S.: $3(-5) = -15$ \hspace{1cm} R.S.: $2(-2) - 7 = -11$
   Since the L.S. < R.S., the point is not a solution.

   d) $5x - 2y + 8 < 0$ \hspace{1cm} D(6, 7)

   In the inequality, substitute: $x = 6$, $y = 7$
   L.S.: $5(6) - 2(7) + 8 = 24$ \hspace{1cm} R.S. = 0
   Since the L.S. > R.S., the point is not a solution.

4. Match each graph with an inequality below.
   i) $2x + y \leq -2$
   ii) $2x + y > 2$
   iii) $x - 2y < 2$
   iv) $x - 2y \geq -1$

   a)

   The line has slope $-2$
   and $y$-intercept 2, so its
   equation is:
   $y = -2x + 2$, or $2x + y = 2$
   The inequality is:
   $2x + y \geq 2$

   b)

   The line has slope 0.5 and
   $y$-intercept 0.5, so its
   equation is:
   $y = 0.5x + 0.5$, or $x - 2y = -1$
   The inequality is:
   $x - 2y \geq -1$
The line has slope $-2$ and \( y \)-intercept $-2$, so its equation is:
\[ y = -2x - 2, \text{ or } 2x + y = -2 \]
The inequality is:
\[ 2x + y \leq -2 \]

The line has slope $0.5$ and \( y \)-intercept $1$, so its equation is:
\[ y = 0.5x - 1, \text{ or } x - 2y = 2 \]
The inequality is:
\[ x - 2y < 2 \]

5. Write an inequality to describe each graph.

a)
The equation can be written as: \( y = -x + 3 \)
The line is broken, and the shaded region is below the line so an inequality is: \( y < -x + 3 \), or \( x + y < 3 \)

b)
The equation can be written as: \( y = x - 3 \)
The line is solid, and the shaded region is below the line so an inequality is: \( y \leq x - 3 \), or \( x - y \geq 3 \)

c)
The equation can be written as: \( y = x + 3 \)
The line is broken, and the shaded region is above the line so an inequality is: \( y > x + 3 \), or \( x - y < -3 \)

d)
The equation can be written as: \( y = -x - 3 \)
The line is broken, and the shaded region is above the line so the inequality is: \( y > -x - 3 \), or \( x + y > -3 \)
6. Graph each linear inequality.

a) \( y \leq 2x + 5 \)

b) \( y > \frac{1}{3}x + 1 \)

c) \( y < -4x - 4 \)

d) \( y \geq \frac{4}{5}x - 2 \)

Use intercepts to graph the related functions.

When \( x = 0 \), \( y = 5 \)  
When \( y = 0 \), \( x = -2.5 \)  
Draw a solid line. Shade the region below the line.

When \( x = 0 \), \( y = 1 \)  
When \( y = 0 \), \( x = 3 \)  
Draw a broken line. Shade the region above the line.

When \( x = 0 \), \( y = -4 \)  
When \( y = 0 \), \( x = -1 \)  
Draw a broken line. Shade the region below the line.

When \( x = 0 \), \( y = -2 \)  
When \( y = 0 \), \( x = 1.5 \)  
Draw a solid line. Shade the region above the line.
7. Graph each linear inequality. Give the coordinates of 3 points that satisfy the inequality.

a) \(5x + 3y > 15\)

Graph of \(5x + 3y > 15\)

Use intercepts to graph the related functions.

When \(x = 0, y = 5\)
When \(y = 0, x = 3\)
Use \((0, 0)\) as a test point.
L.S. = 0; R.S. = 15
Since \(0 < 15\), the origin does not lie in the shaded region.
Draw a broken line. Shade the region above the line.
From the graph, 3 points that satisfy the inequality are: \((2, 3), (1, 5), (3, 2)\)

b) \(3x - 2y \leq -9\)

Graph of \(3x - 2y \leq -9\)

Use intercepts to graph the related functions.

When \(x = 0, y = 4.5\)
When \(y = 0, x = -3\)
Use \((0, 0)\) as a test point.
L.S. = 0; R.S. = -9
Since \(0 > -9\), the origin lies in the shaded region.
Draw a solid line. Shade the region above the line.
From the graph, 3 points that satisfy the inequality are: \((-2, 3), (-1, 4), (-1, 6)\)

c) \(x + 6y \geq -4\)

Graph of \(x + 6y \geq -4\)

Graph the related functions.

When \(y = 0, x = -4\)
When \(y = -1, x = 2\)
Use \((0, 0)\) as a test point.
L.S. = 0; R.S. = -4
Since \(0 > -4\), the origin lies in the shaded region.
Draw a solid line. Shade the region above the line.
From the graph, 3 points that satisfy the inequality are: \((2, 1), (1, 2), (3, 3)\)

d) \(4x - 7y < 21\)

Graph of \(4x - 7y < 21\)

Graph the related functions.

When \(x = 0, y = -3\)
When \(y = 1, x = 7\)
Use \((0, 0)\) as a test point.
L.S. = 0; R.S. = 21
Since \(0 < 21\), the origin lies in the shaded region.
Draw a solid line. Shade the region above the line.
From the graph, 3 points that satisfy the inequality are: \((-1, 3), (1, -1), (2, 3)\)
8. Write an inequality to describe each graph.

a) The line has slope $-2$ and $y$-intercept 1, so its equation is: $y = -2x + 1$
   The line is solid and the region below is shaded.
   An inequality is: $y \leq -2x + 1$

b) The line has slope $\frac{3}{4}$ and $y$-intercept 5, so its equation is: $y = \frac{3}{4}x + 5$
   The line is broken and the region below is shaded.
   An inequality is: $y < \frac{3}{4}x + 5$

9. A student graphed the inequality $2x - y < 0$ and used the origin as a test point. Could the student then shade the correct region of the graph? Explain your answer.

No, the line passes through the origin, so it cannot be used as a test point. The test point must not lie on the line that divides the region.

10. Use technology to graph each linear inequality. Sketch the graph.

a) $y < 1.6x - 1.95$
   Graph: $y = 1.6x - 1.95$
   The boundary is not part of the graph.

b) $y > \frac{4}{3}x + \frac{3}{7}$
   Graph: $y = \frac{4}{3}x + \frac{3}{7}$
   The boundary is not part of the graph.

c) $8x - 3y - 25 \geq 0$
   $3y \leq 8x - 25$
   $y \leq \frac{8}{3}x - \frac{25}{3}$
   Graph: $y = \frac{8}{3}x - \frac{25}{3}$
   The boundary is part of the graph.

d) $4.8x + 2.3y - 3.7 \leq 0$
   $2.3y \leq -4.8x + 3.7$
   $y \leq -\frac{4.8}{2.3}x + \frac{3.7}{2.3}$
   Graph: $y = -\frac{4.8}{2.3}x + \frac{3.7}{2.3}$
   The boundary is part of the graph.
11. Nina takes her friends to an ice cream store. A milkshake costs $3 and a chocolate sundae costs $2.50. Nina has $18 in her purse.

a) Write an inequality to describe how Nina can spend her money.

Let \( m \) represent the number of milkshakes and \( s \) represent the number of sundaes.

An inequality is: \( 3m + 2.5s \leq 18 \)

b) Determine 3 possible ways Nina can spend up to $18.

Determine the coordinates of 2 points that satisfy the related function.

When \( s = 0 \), \( m = 6 \)

When \( s = 6 \), \( m = 1 \)

Join the points with a solid line.

The solution is the points, with whole-number coordinates, on and below the line.

Three ways are: 4 milkshakes, 2 sundaes; 3 milkshakes, 3 sundaes; 2 milkshakes, 4 sundaes

The point, with whole-number coordinates, that is closest to the line has coordinates (5, 1); the cost, in dollars, is:

\[ (5)(3) + (1)(2.50) = 17.50 \]

Nina can spend $17.50 and still have change.

c) What is the most money Nina can spend and still have change from $18?

The point, with whole-number coordinates, that is closest to the line has coordinates (5, 1); the cost, in dollars, is:

\[ (5)(3) + (1)(2.50) = 17.50 \]

Nina can spend $17.50 and still have change.

12. The relationship between two negative numbers \( p \) and \( q \) is described by the inequality \( p - 2q > -6 \).

a) What are the restrictions on the variables?

Since the numbers are negative, \( p < 0 \) and \( q < 0 \)

b) Graph the inequality.

Determine the coordinates of 2 points that satisfy the related function.

When \( p = -10 \), \( q = -2 \)

When \( p = -6 \), \( q = 0 \)

Draw a broken line through the points.

The solution is the points below the line in Quadrant 3.

The point, with whole-number coordinates, that is closest to the line has coordinates (−4, −4) and (−12, −4)
13. Graph each inequality for the given restrictions on the variables.

a) \( y > -3x + 4; \) for \( x > 0, y > 0 \)

Since \( x > 0, y > 0 \), the graph is in Quadrant 1.
The graph of the related function has slope \(-3\) and \(y\)-intercept \(4\).
Draw a broken line to represent the related function in Quadrant 1.
Shade the region above the line.
The axes bounding the graph are broken lines.

b) \( 2x - 3y < 6; \) for \( x \geq 0, y \leq 0 \)

Since \( x \geq 0, y \leq 0 \), the graph is in Quadrant 4.
Graph the related function.
When \( y = 0, x = 3 \)
When \( x = 0, y = -2 \)
Draw a broken line in Quadrant 4.
Use \((0, 0)\) as a test point.
L.S. = 0; R.S. = 6
Since \( 0 < 6 \), the origin lies in the shaded region.
Shade the region above the line.

c) \( 4x + 5y - 20 > 0; \) for \( x \leq 0, y \geq 0 \)

Since \( x \leq 0, y \geq 0 \), the graph is in Quadrant 2.
Graph the related function.
When \( x = 0, y = 4 \)
When \( x = -5, y = 8 \)
Draw a broken line in Quadrant 2.
Use \((0, 0)\) as a test point.
L.S. = \(-20\); R.S. = 0
Since \(-20 < 0\), the origin does not lie in the shaded region.
Shade the region above the line.

14. a) For \( A(9, a) \) to be a solution of \( 3x - 2y < 5 \), what must be true about \( a \)?

Substitute the coordinates of \( A \) in the inequality.
\[ 3(9) - 2(a) < 5 \]
Solve for \( a \).
\[ 2a > 22 \]
\[ a > 11 \]

b) For \( B(b, -3) \) to be a solution of \( 3x + 4y \geq -12 \), what must be true about \( b \)?

Substitute the coordinates of \( B \) in the inequality.
\[ 3(b) + 4(-3) \geq -12 \]
Solve for \( b \).
\[ 3b \geq 0 \]
\[ b \geq 0 \]
15. A personal trainer books clients for either 45-min or 60-min appointments. He meets with clients a maximum of 40 h each week.

a) Write an inequality that represents the trainer’s weekly appointments.

Let \( x \) represent the number of 45-min appointments and \( y \) represent the number of 60-min appointments.

An inequality is: \( 45x + 60y \leq 2400 \)

Divide by 15.

\( 3x + 4y \leq 160 \)

b) Graph the related equation, then describe the graph of the inequality.

Determine the coordinates of 2 points that satisfy the related function.

When \( x = 0, y = 40 \)
When \( x = 20, y = 25 \)

Join the points with a solid line.

The solution is the points, with whole-number coordinates, on and below the line.

For no 60-min appointments, \( y = 0 \), so the point is on the \( x \)-axis; it is the point with whole-number coordinates that is closest to the \( x \)-intercept of the graph of the related equation. When \( y = 0 \),

\( x = \frac{2400}{45} \), or \( 53.3 \)

Fifty-three 45-min appointments are possible.

16. Graph this inequality. Identify the strategy you used and explain why you chose that strategy.

\( \frac{x}{3} + \frac{y}{2} \geq 1 \)

Graph the related function.

Determine the intercepts.

When \( y = 0, x = 3 \)
When \( x = 0, y = 2 \)

Draw a solid line.

Use \( (0, 0) \) as a test point.

L.S. = 0; R.S. = 1

Since \( 0 < 1 \), the origin is not in the shaded region.

Shade the region above the line.
17. How is a linear inequality in two variables similar to a linear inequality in one variable? How are the inequalities different?

The solutions of both inequalities are usually sets of values. A linear inequality in one variable is a set of numbers that can be represented on a number line. A linear inequality in two variables is a set of ordered pairs that can be represented on a coordinate plane.