Lesson 4.2 Math Lab: Assess Your Understanding, pages 265–267

1. Use the graph of $y = x^2 - 4x - 12$ to determine the roots of $x^2 - 4x - 12 = 0$. Explain your strategy.

The roots of the equation are the $x$-intercepts of the graph. These are: $x = -2$ and $x = 6$
2. Use graphing technology to solve each equation. Verify the solution.
   a) \(-4x^2 - 13x - 12 = 0\)  
   b) \(-2x^2 - 9x + 35 = 0\)

   a) For \(-4x^2 - 13x - 12 = 0\), graph \(y = -4x^2 - 13x - 12\). The graph does not intersect the \(x\)-axis, so the equation \(-4x^2 - 13x - 12 = 0\) does not have any real roots.

   b) For \(-2x^2 - 9x + 35 = 0\), graph \(y = -2x^2 - 9x + 35\). On a graphing calculator, press: \(\text{2nd TRACE} [2]\). Move the cursor to the left of the 1st \(x\)-intercept, then press \(\text{ENTER}\); move the cursor to the right of the intercept and press \(\text{ENTER} \text{ ENTER}\). The screen displays \(X = -7\). Repeat the process for the 2nd \(x\)-intercept to get \(X = 2.5\). The roots are \(x = -7\) and \(x = 2.5\).

3. Use graphing technology to approximate the solution of each equation. Write the roots to 1 decimal place.
   a) \(x^2 - 3x + 1 = 0\)  
   b) \(-x^2 + 7x - 1 = 0\)

   a) For \(x^2 - 3x + 1 = 0\), graph \(y = x^2 - 3x + 1\). Use the strategy of question 2b to display \(X = 3.8196601\) and \(X = 2.618034\). The roots are approximately \(x = 0.4\) and \(x = 2.6\).

   b) For \(-x^2 + 7x - 1 = 0\), graph \(y = -x^2 + 7x - 1\). Use the strategy of question 2b to display \(X = 0.14589803\) and \(X = 6.854102\). The roots are approximately \(x = 0.1\) and \(x = 6.9\).

   c) \(2x^2 + 6x - 3 = 0\)  
   d) \(-3x^2 - 7x + 1 = 0\)

   c) For \(2x^2 + 6x - 3 = 0\), graph \(y = 2x^2 + 6x - 3\). Use the strategy of question 2b to display \(X = -3.436492\) and \(X = 0.43649167\). The roots are approximately \(x = -3.4\) and \(x = 0.4\).

   d) For \(-3x^2 - 7x + 1 = 0\), graph \(y = -3x^2 - 7x + 1\). Use the strategy of question 2b to display \(X = -2.468375\) and \(X = 0.13504161\). The roots are approximately \(x = -2.5\) and \(x = 0.1\).

4. The graph of a quadratic function is shown. What can you say about the discriminant of the corresponding quadratic equation? Justify your response.

   The graph intersects the \(x\)-axis in two points, so the related quadratic equation has 2 real roots. This means that the discriminant is greater than 0. In the quadratic formula, \(x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\), there are two values for \(x\) because the square root of the discriminant is added to and subtracted from \(-b\).
5. a) Sketch graphs to show why a quadratic equation may have 1, 2, or no real roots. Explain why a quadratic equation cannot have 3 or more roots.

This graph touches the x-axis at 1 point, so the related quadratic equation has 1 real root.

This graph intersects the x-axis at 2 points, so the related quadratic equation has 2 real roots.

This graph does not intersect the x-axis, so the related quadratic equation has no real roots.

A quadratic equation cannot have 3 or more roots because its related quadratic function intersects the x-axis in no more than 2 points.

b) How can the discriminant be used to determine the number of roots of a quadratic equation?

When the discriminant is 0, there is exactly 1 real root.
When the discriminant is positive, there are 2 real roots.
When the discriminant is negative, there are no real roots.