Lesson 3.4 Exercises, pages 217–226

A

4. Identify the values of $a$, $b$, and $c$ to make each quadratic equation match the general form $ax^2 + bx + c = 0$.

a) $x^2 + 9x - 2 = 0$  
   Compare each equation to $ax^2 + bx + c = 0$  
   $a = 1, b = 9, c = -2$

b) $4x^2 - 11x = 0$  
   $a = 4, b = -11, c = 0$

c) $11x - 3x^2 + 8 = 0$  
   $a = -3, b = 11, c = 8$

d) $3.2x^2 + 6.1 = 0$  
   $a = 3.2, b = 0, c = 6.1$
5. Simplify each radical expression.
   a) \[
   \frac{6 \pm \sqrt{16}}{2} = \frac{6 \pm 4}{2} = 5; \text{ or } \frac{6 - 4}{2} = 1
   \]
   b) \[
   \frac{-8 \pm \sqrt{80}}{4} = \frac{-8 \pm 4\sqrt{5}}{4}
   \]
   c) \[
   \frac{3 \pm \sqrt{45}}{6} = \frac{3 \pm 3\sqrt{5}}{6} = \frac{3(1 \pm \sqrt{5})}{6} = \frac{1 \pm \sqrt{5}}{2}
   \]
   d) \[
   \frac{12 \pm \sqrt{28}}{4} = \frac{12 \pm 2\sqrt{7}}{4}
   \]

6. Solve each quadratic equation.
   a) \[x^2 + 6x + 4 = 0\]
   b) \[x^2 - 10x + 17 = 0\]

   For each equation, substitute for \(a\), \(b\), and \(c\) in:
   \[x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\]

   a) \(a = 1, b = -6, c = 4\)
   \[x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(4)}}{2(1)} = \frac{6 \pm \sqrt{20}}{2} = \frac{6 \pm 2\sqrt{5}}{2} = 3 \pm \sqrt{5}\]

   b) \(a = 1, b = -10, c = 17\)
   \[x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(17)}}{2(1)} = \frac{10 \pm \sqrt{100 - 68}}{2} = \frac{10 \pm \sqrt{32}}{2} = \frac{10 \pm 4\sqrt{2}}{2} = 5 \pm 2\sqrt{2}\]

   c) \[x^2 + 4x - 3 = 0\]
   \(a = 1, b = 4, c = -3\)
   \[x = \frac{-4 \pm \sqrt{4^2 - 4(1)(-3)}}{2(1)} = \frac{-4 \pm \sqrt{16 + 12}}{2} = \frac{-4 \pm \sqrt{28}}{2} = \frac{-4 \pm 2\sqrt{7}}{2} = -2 \pm \sqrt{7}\]

   d) \[2x^2 - 2x - 1 = 0\]
   \(a = 2, b = -2, c = -1\)
   \[x = \frac{2 \pm \sqrt{(-2)^2 - 4(2)(-1)}}{2(2)} = \frac{2 \pm \sqrt{4 + 8}}{4} = \frac{2 \pm \sqrt{12}}{4} = \frac{2 \pm 2\sqrt{3}}{4} = \frac{1 \pm \sqrt{3}}{2}\]
7. Solve each quadratic equation.

a) $3x^2 = 4x + 1$

For each equation, substitute for $a$, $b$, and $c$ in:

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$3x^2 - 4x - 1 = 0$

$a = 3, b = -4, c = -1$

$x = \frac{4 \pm \sqrt{(-4)^2 - 4(3)(-1)}}{2(3)}$

$x = \frac{4 \pm \sqrt{28}}{6}$

$x = \frac{2 \pm \sqrt{7}}{3}$

b) $4x^2 - 1 = -7x$

$a = 4, b = 7, c = -1$

$x = \frac{-7 \pm \sqrt{7^2 - 4(-1)}}{2(4)}$

$x = \frac{-7 \pm \sqrt{65}}{8}$

$c) 2x(x - 3) = 4(x - 3) + 1$

$2x^2 - 6x - 4x + 12 - 1 = 0$

$2x^2 - 10x + 11 = 0$

$a = 2, b = -10, c = 11$

$x = \frac{10 \pm \sqrt{(-10)^2 - 4(2)(11)}}{2(2)}$

$x = \frac{10 \pm \sqrt{12}}{4}$

$x = \frac{5 \pm \sqrt{3}}{2}$

d) $(2x + 1)^2 + 2 = 0$

$x = \frac{-4 \pm \sqrt{-32}}{8}$

The radicand is negative, so there are no real roots.

8. A student wrote the solution below to solve this quadratic equation:

$2x^2 - 3 = 7x$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(2)(-3)}}{2(2)}$

$x = \frac{7 \pm \sqrt{73}}{4}$

Identify the error, then write the correct solution.

The student wrote an incorrect quadratic formula.
The correct solution is:

$x = \frac{7 \pm \sqrt{(-7)^2 - 4(2)(-3)}}{2(2)}$

$x = \frac{7 \pm \sqrt{73}}{4}$
9. a) Solve each equation by factoring.
   
   i) \(3x^2 = 11x + 20\)
   \[3x^2 - 11x - 20 = 0\]
   \((3x + 4)(x - 5) = 0\)
   \(x = \frac{-4}{3}\) or \(x = 5\)

   ii) \(12x^2 + 8x = 15\)
   \[12x^2 + 8x - 15 = 0\]
   \((2x + 3)(6x - 5) = 0\)
   \(x = -\frac{3}{2}\) or \(x = \frac{5}{6}\)

b) Solve each equation in part a using the quadratic formula.

   For each equation, substitute for \(a\), \(b\), and \(c\) in:
   \[x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\]

   i) \(3x^2 - 11x - 20 = 0\)
   \[a = 3, b = -11, c = -20\]
   \[x = \frac{11 \pm \sqrt{(-11)^2 - 4(3)(-20)}}{2(3)}\]
   \[x = \frac{11 \pm \sqrt{361}}{6}\]
   \[x = \frac{11 \pm 19}{6}\]
   \[x = \frac{11 + 19}{6} = \frac{30}{6} = 5\]
   \[x = \frac{11 - 19}{6} = \frac{-8}{6} = -\frac{4}{3}\]
   \(\text{Or, } x = \frac{11 - 19}{6} = -\frac{4}{3}\)

   ii) \(12x^2 + 8x - 15 = 0\)
   \[a = 12, b = 8, c = -15\]
   \[x = \frac{-8 \pm \sqrt{8^2 - 4(12)(-15)}}{2(12)}\]
   \[x = \frac{-8 \pm \sqrt{64 + 720}}{24}\]
   \[x = \frac{-8 \pm \sqrt{784}}{24}\]
   \[x = \frac{-8 \pm 28}{24}\]
   \[x = \frac{-8 + 28}{24} = \frac{20}{24} = \frac{5}{6}\]
   \[x = \frac{-8 - 28}{24} = \frac{-36}{24} = -\frac{3}{2}\]

   \(\text{Or, } x = \frac{-8 - 28}{24} = -\frac{3}{2}\)

   c) Which method do you prefer and why?

   I prefer to factor when the numbers are small because it is quicker.
   I prefer to use the quadratic formula when the numbers are large
   and I have many factors to guess and test.

10. For each equation, choose a solution strategy, justify your choice,
    then solve the equation.

   a) \(2x^2 + 9x + 8 = 0\)
   b) \(x^2 + 7x - 30 = 0\)

   I use the formula because I cannot factor.
   I can factor.
   \(x = 3\) or \(x = -10\)

   \(\text{Substitute: } a = 2, b = 9, c = 8\)
   \[(x - 3)(x + 10) = 0\]

   \(x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\)
   \[x = \frac{-9 \pm \sqrt{9^2 - 4(2)(8)}}{2(2)}\]
   \[x = \frac{-9 \pm \sqrt{81 - 64}}{4}\]
   \[x = \frac{-9 \pm 7}{4}\]
   \[x = \frac{-9 + 7}{4} = \frac{-2}{4} = -\frac{1}{2}\]
   \[x = \frac{-9 - 7}{4} = \frac{-16}{4} = -4\]
c) \((x + 6)^2 = 12\)

\[x^2 + 12x + 24 = 0\]

I use the formula because I cannot factor. Substitute:

\[a = 1, \ b = 12, \ c = 24\]

\[in: \ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\]

\[x = \frac{-12 \pm \sqrt{12^2 - 4(1)(24)}}{2(1)}\]

\[x = \frac{-12 \pm \sqrt{48}}{2}\]

\[x = \frac{-12 \pm 4\sqrt{3}}{2}\]

\[x = -6 \pm 2\sqrt{3}\]

d) \(8 + 5.6x - 1.2x^2 = 0\)

I use the formula because I cannot factor. Substitute:

\[a = -1.2, \ b = 5.6, \ c = 8\]

\[in: \ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\]

\[x = \frac{-5.6 \pm \sqrt{5.6^2 - 4(-1.2)(8)}}{2(-1.2)}\]

\[x = \frac{-5.6 \pm \sqrt{69.76}}{-2.4}\]

\[x = \frac{5.6 \pm 8.92}{-2.4}\]

There are no real roots.

11. Solve each quadratic equation. Give the solution to 3 decimal places.

a) \(\frac{1}{3}x^2 - 3x + \frac{1}{4} = 0\)

Multiply by 12.

\[4x^2 - 36x + 3 = 0\]

Substitute:

\[a = 4, \ b = -36, \ c = 3\]

\[in: \ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\]

\[x = \frac{36 \pm \sqrt{(-36)^2 - 4(4)(3)}}{2(4)}\]

\[x = \frac{36 \pm \sqrt{1248}}{8}\]

\[x = \frac{36 + \sqrt{1248}}{8}\]

so, \(x \approx 8.916\)

Or, \(x = \frac{36 - \sqrt{1248}}{8}\)

so, \(x \approx 0.084\)

b) \(-2x^2 + \frac{3}{2}x - \frac{4}{5} = 0\)

Multiply by 10.

\[-20x^2 + 15x - 8 = 0\]

Substitute:

\[a = -20, \ b = 15, \ c = -8\]

\[in: \ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\]

\[x = \frac{-15 \pm \sqrt{15^2 - 4(-20)(-8)}}{2(-20)}\]

\[x = \frac{-15 \pm \sqrt{-415}}{-40}\]

There are no real roots.

c) \(4.9x^2 + 12x - 0.8 = 0\)

Substitute:

\[a = 4.9, \ b = 12, \ c = -0.8\]

\[in: \ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\]

\[x = \frac{-12 \pm \sqrt{12^2 - 4(4.9)(-0.8)}}{2(4.9)}\]

\[x = \frac{-12 \pm \sqrt{159.68}}{9.8}\]

\[x = \frac{-12 + \sqrt{159.68}}{9.8}\]

so, \(x \approx 0.065\)

Or, \(x = \frac{-12 - \sqrt{159.68}}{9.8}\)

so, \(x \approx -2.514\)

d) \(2.1x^2 = 1.2x + 3\)

Substitute:

\[a = 2.1, \ b = -1.2, \ c = -3\]

\[in: \ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\]

\[x = \frac{1.2 \pm \sqrt{(-1.2)^2 - 4(2.1)(-3)}}{2(2.1)}\]

\[x = \frac{1.2 \pm \sqrt{26.64}}{4.2}\]

so, \(x \approx 1.515\)

Or, \(x = \frac{1.2 - \sqrt{26.64}}{4.2}\)

so, \(x \approx -0.943\)
12. Solve each radical equation. Check for extraneous roots.
   a) \(2 + \sqrt{5x} = 3x\)
   b) \(2x = \sqrt{2x + 10} - 3\)

   \[
   \sqrt{5x} = 3x - 2 \\
   (\sqrt{5x})^2 = (3x - 2)^2 \\
   5x = 9x^2 - 12x + 4 \\
   \]
   \[
   9x^2 - 17x + 4 = 0 \\
   \]
   Substitute: 
   \[
   a = 9, b = -17, c = 4 \\
   \]
   in: 
   \[
   x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
   x = \frac{17 \pm \sqrt{(-17)^2 - 4(9)(4)}}{2(9)} \\
   x = \frac{17 \pm \sqrt{145}}{18} \\
   \]
   Use a calculator to check:
   The root is: \(x = \frac{17 + \sqrt{145}}{18}\)

   \[
   2x + 3 = \sqrt{2x + 10} \\
   (2x + 3)^2 = (\sqrt{2x + 10})^2 \\
   4x^2 + 12x + 9 = 2x + 10 \\
   \]
   \[
   4x^2 + 10x - 1 = 0 \\
   \]
   Substitute: 
   \[
   a = 4, b = 10, c = -1 \\
   \]
   in: 
   \[
   x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
   x = \frac{-10 \pm \sqrt{10^2 - 4(4)(-1)}}{2(4)} \\
   x = \frac{-10 \pm \sqrt{116}}{8} \\
   x = \frac{-10 \pm 2\sqrt{29}}{8} \\
   x = \frac{5 \pm \sqrt{29}}{4} \\
   \]
   Use a calculator to check:
   The root is: \(x = \frac{5 + \sqrt{29}}{4}\)

13. a) Solve this equation using each strategy below: \(x^2 - 10x - 24 = 0\)
   i) the quadratic formula
   \[
   x^2 - 10x - 24 = 0 \\
   \]
   Substitute: 
   \[
   a = 1, b = -10, c = -24 \\
   \]
   in: 
   \[
   x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
   x = \frac{10 \pm \sqrt{(-10)^2 - 4(1)(-24)}}{2(1)} \\
   x = \frac{10 \pm \sqrt{196}}{2} \\
   x = \frac{10 \pm 14}{2} \\
   x = \frac{10 + 14}{2} = 12 \\
   \]
   Or, \(x = \frac{10 - 14}{2} = -2\)

   ii) completing the square
   \[
   x^2 - 10x - 24 = 0 \\
   \]
   \[
   x^2 - 10x + 25 = 24 + 25 \\
   (x - 5)^2 = 49 \\
   x - 5 = \pm \sqrt{49} \\
   x = 5 \pm 7 \\
   x = 12 \text{ or } x = -2 \\
   \]

   iii) factoring
   \[
   x^2 - 10x - 24 = 0 \\
   (x - 12)(x + 2) = 0 \\
   x = 12 \text{ or } x = -2 \\
   \]

b) Which strategy do you prefer? Is it the most efficient? Explain.

   Sample response: I prefer factoring; it is the most efficient because it takes less time and less space.
14. A person is standing on a bridge over a river. She throws a pebble upward. The height of the pebble above the river, $h$ metres, is given by the formula $h = 26 + 9t - 4.9t^2$, where $t$ is the time in seconds after the pebble is thrown.

a) When will the pebble be 20 m above the river? Give the answer to the nearest tenth of a second.

In $h = 26 + 9t - 4.9t^2$, substitute $h = 20$, then solve for $t$.

$20 = 26 + 9t - 4.9t^2$
$0 = 6 + 9t - 4.9t^2$

Substitute: $a = -4.9$, $b = 9$, $c = 6$ in: $t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$t = \frac{-9 \pm \sqrt{9^2 - 4(-4.9)(6)}}{2(-4.9)}$

$t = \frac{9 \pm \sqrt{198.6}}{9.8}$

Ignore the negative root since $t$ cannot be negative.

$t = \frac{9 + \sqrt{198.6}}{9.8}$
$t = 2.3563...$

The pebble is 20 m above the river after approximately 2.4 s.

b) When will the pebble be 30 m above the river? Give the answer to the nearest tenth of a second.

In $h = 26 + 9t - 4.9t^2$, substitute $h = 30$, then solve for $t$.

$30 = 26 + 9t - 4.9t^2$
$0 = -4 + 9t - 4.9t^2$

Substitute: $a = -4.9$, $b = 9$, $c = -4$ in: $t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$t = \frac{-9 \pm \sqrt{9^2 - 4(-4.9)(-4)}}{2(-4.9)}$

$t = \frac{9 \pm \sqrt{2.6}}{9.8}$

$t = \frac{9 + \sqrt{2.6}}{9.8}$, or 1.0829...

$t = \frac{9 - \sqrt{2.6}}{9.8}$, or 0.7538...

The pebble is 30 m above the river after approximately 0.8 s and 1.1 s.

c) Why are there two answers for part b, but only one answer for part a?

There are two answers for part b because the stone is 30 m above the river on its way up and on its way down. There is only one answer for part a because the stone is only 20 m above the river on its way down.
15. A car was travelling at a constant speed of 19 m/s, then accelerated for 10 s. The distance travelled during this time, \(d\) metres, is given by the formula \(d = 19t + 0.7t^2\), where \(t\) is the time in seconds since the acceleration began. How long did it take the car to travel 200 m? Give the answer to the nearest tenth of a second.

\[
\begin{align*}
\text{In } d &= 19t + 0.7t^2, \text{ substitute } d = 200, \text{ then solve for } t. \\
200 &= 19t + 0.7t^2 \\
0 &= -200 + 19t + 0.7t^2 \\
\text{Substitute: } a &= 0.7, b = 19, c = -200 \text{ in: } t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
&= \frac{-19 \pm \sqrt{19^2 - 4(0.7)(-200)}}{2(0.7)} \\
&= \frac{-19 \pm \sqrt{921}}{1.4} \\
&= \frac{-19 + \sqrt{921}}{1.4} \\
&= 8.1057\ldots
\end{align*}
\]

The car travelled 200 m in approximately 8.1 s.

16. Josie’s rectangular garden measures 9 m by 13 m. She wants to double the area of her garden by adding equal lengths to both dimensions. Determine this length to the nearest centimetre.

Let the length added be \(x\) metres.

The new width, in metres, is: \(x + 9\)
The new length, in metres, is: \(x + 13\)
The new area, in square metres is: \((x + 9)(x + 13)\)
The original area is: \((9)(13)\), or 117 m\(^2\)
The new area is: \(2(117) = 234\) m\(^2\)

An equation is: \((x + 9)(x + 13) = 234\)
\[x^2 + 22x + 117 = 234\]
\[x^2 + 22x - 117 = 0\]

Substitute: \(a = 1, b = 22, c = -117\) in: \(x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\)
\[x = \frac{-22 \pm \sqrt{22^2 - 4(1)(-117)}}{2(1)}\]
\[x = \frac{-22 \pm \sqrt{952}}{2}\]

Ignore the negative root since \(x\) cannot be negative.
\[x = \frac{-22 + \sqrt{952}}{2}\]
\[x = 4.4272\ldots\]
The length added is approximately 4.43 m.
17. a) Solve this equation \( \frac{1}{2}x^2 - \frac{3}{4}x - 1 = 0 \) in the two ways described below:

i) Substitute the given coefficients and constant in the quadratic formula.

\[
\frac{1}{2}x^2 - \frac{3}{4}x - 1 = 0
\]

Substitute: \( a = \frac{1}{2}, \ b = -\frac{3}{4}, \ c = -1 \) in:

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[
x = \frac{3}{4} \pm \sqrt{\left(-\frac{3}{4}\right)^2 - 4\left(\frac{1}{2}\right)(-1)}
\]

\[
x = \frac{3}{4} \pm \sqrt{\frac{41}{16}}
\]

\[
x = \frac{3}{4} \pm \frac{\sqrt{41}}{4}
\]

ii) Multiply the equation by a common denominator to remove the fractions, then substitute in the quadratic formula.

\[
\frac{1}{2}x^2 - \frac{3}{4}x - 1 = 0 \quad \text{Multiply by 4.}
\]

\[
2x^2 - 3x - 4 = 0
\]

Substitute: \( a = 2, \ b = -3, \ c = -4 \) in:

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[
x = \frac{3 \pm \sqrt{(-3)^2 - 4(2)(-4)}}{2(2)}
\]

\[
x = \frac{3 \pm \sqrt{49}}{4}
\]

b) Which strategy in part a do you prefer? Explain why.

I prefer the strategy in part ii because it is easier to work with integers than fractions.

18. This quadratic equation has only one root: \( 2x^2 + 6x + d = 0 \)

Use the quadratic formula to determine the value of \( d \). Explain your strategy.

\[
2x^2 + 6x + d = 0
\]

Substitute: \( a = 2, \ b = 6, \ c = d \) in:

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[
x = \frac{-6 \pm \sqrt{6^2 - 4(2)(d)}}{2(2)}
\]

The equation has only one root, so the radicand must be 0.

\[
36 - 8d = 0
\]

\[
d = 4.5
\]
19. a) Solve this quadratic equation by expanding, simplifying, then applying the quadratic formula: \(2(x - 5)^2 - 7(x - 5) - 2 = 0\)

\[
2x^2 - 20x + 50 - 7x + 35 - 2 = 0 \\
2x^2 - 27x + 83 = 0
\]

Substitute: \(a = 2, b = -27, c = 83\) in: \(x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\)

\[
x = \frac{27 \pm \sqrt{(-27)^2 - 4(2)(83)}}{2(2)} \\
x = \frac{27 \pm \sqrt{65}}{4}
\]

b) Solve the equation in part a using the quadratic formula without expanding.

\(2(x - 5)^2 - 7(x - 5) - 2 = 0\)

Substitute: \(a = 2, b = -7, c = -2\) in: \(x - 5 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\)

\[
x - 5 = \frac{7 \pm \sqrt{(-7)^2 - 4(2)(-2)}}{4} \\
x - 5 = \frac{7 \pm \sqrt{65}}{4} \\
x = 5 + \frac{7 \pm \sqrt{65}}{4} \\
x = \frac{27 \pm \sqrt{65}}{4}
\]

20. a) Is this equation quadratic: \(x^4 + x^2 = 1\)? Justify your response.

The equation is not quadratic because it contains an \(x^4\)-term.

b) Describe a strategy you could use to solve the equation in part a.

Write the equation as: \((x^2)^2 + (x^2) - 1 = 0\), then use the quadratic formula.

c) Solve the equation in part a.

Substitute: \(a = 1, b = 1, c = -1\) in: \(x^2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\)

\[
x^2 = \frac{-1 \pm \sqrt{1^2 - 4(1)(-1)}}{2(1)} \\
x^2 = \frac{-1 \pm \sqrt{5}}{2} \\
\text{Since } x^2 \text{ is positive, ignore the negative root.} \\
x = \pm \sqrt{\frac{-1 + \sqrt{5}}{2}}
\]