Lesson 2.3 Exercises, pages 114–119

A

3. a) Simplify each radical, if possible.

\[ \sqrt{27} = \sqrt{9 \cdot 3} = 3 \sqrt{3} \quad 2\sqrt{2} \quad \sqrt{8} = \sqrt{4 \cdot 2} = 2\sqrt{2} \quad \sqrt{32} = \sqrt{16 \cdot 2} = 4\sqrt{2} \]

\[ \sqrt{6} \quad 2\sqrt[3]{3} \quad \sqrt{48} \quad \sqrt{18} = \sqrt{16 \cdot 3} = 4\sqrt{3} \quad \sqrt{18} = \sqrt{9 \cdot 2} = 3\sqrt{2} \]

b) Group the radicals in part a into sets of like radicals.

All radicals have index 2.
Radicals with radicand 2: \(3\sqrt{2}, 2\sqrt{2}, 4\sqrt{2}, 3\sqrt{2}\); that is, \(3\sqrt{2}, \sqrt{8}, \sqrt{32}, \sqrt{18}\)
Radicals with radicand 3: \(3\sqrt{3}, 2\sqrt[3]{3}, 4\sqrt[3]{3}\); that is, \(\sqrt{27}, 2\sqrt[3]{3}, \sqrt{48}\)
4. Simplify by adding or subtracting like terms.
   a) $3\sqrt{2} + 2\sqrt{2} - 5\sqrt{2} = (3 + 2 - 5)\sqrt{2} = 0$
   b) $\sqrt{108} - 2\sqrt{3} - \sqrt{75} = \sqrt{36 \cdot 3} - 2\sqrt{3} - \sqrt{25 \cdot 3} = 6\sqrt{3} - 2\sqrt{3} - 5\sqrt{3} = -\sqrt{3}$
   c) $5\sqrt{7} + 2\sqrt{5} - \sqrt{28} + \sqrt{45} = 5\sqrt{7} + 2\sqrt{5} - \sqrt{4 \cdot 7} + \sqrt{9 \cdot 5} = 5\sqrt{7} + 2\sqrt{5} - 2\sqrt{7} + 3\sqrt{5} = 3\sqrt{7} + 5\sqrt{5}$
   d) $\sqrt[4]{16} + \sqrt[3]{75} - 3\sqrt[2]{2} = \sqrt[4]{2^4} + \sqrt[3]{3^2 \cdot 5^2} - 3\sqrt[2]{2} = 2\sqrt[4]{2} + 5\sqrt[3]{3} - 3\sqrt[2]{2} = -\sqrt[2]{2} + 5\sqrt[3]{3}$

5. Simplify.
   a) $6\sqrt{x} - 4\sqrt{x} + 2\sqrt{x} + \sqrt{x}, x \geq 0$
      $= (6 - 4 + 2 + 1)\sqrt{x} = 5\sqrt{x}$
   b) $\sqrt{4a} + \sqrt{16a} - \sqrt{9a}, a \geq 0$
      $= \sqrt{4 \cdot a} + \sqrt{16 \cdot a} - \sqrt{9 \cdot a} = 2\sqrt{a} + 4\sqrt{a} - 3\sqrt{a} = 3\sqrt{a}$
   c) $\sqrt[3]{27x^3} - \sqrt[3]{8x^3} + \sqrt[3]{64x^3}, x \in \mathbb{R}$
      $= \sqrt[3]{27} \cdot x - \sqrt[3]{8} \cdot x + \sqrt[3]{64} \cdot x^2 = 3\sqrt[3]{x} - 2\sqrt[3]{x} + 4\sqrt[3]{x^2} = 5\sqrt[3]{x^2}$

6. Explain why it is necessary to write $\sqrt[4]{x}$ as $|x|$.
   $\sqrt[4]{x}$ is defined for $x \in \mathbb{R}$. The radical sign indicates the principal root, so the value of $\sqrt[4]{x}$ cannot be negative. Although $x^4$ is always positive or zero, $x$ can be negative. So, it is necessary to write $\sqrt[4]{x}$ as $|x|$.

7. Identify the values of the variables for which each radical is defined, then simplify.
   a) $7\sqrt{-x} + 15\sqrt{-x} - 13\sqrt{-x}$
      The radicand cannot be negative, so $-x \geq 0$; that is, $x \leq 0$.
      $7\sqrt{-x} + 15\sqrt{-x} - 13\sqrt{-x} = (7 + 15 - 13)\sqrt{-x} = 9\sqrt{-x}$
b) \( \sqrt{28mn^2} + m^n \sqrt{63n} \)

The radicand cannot be negative.

Since \( m^n \geq 0, \ m \in \mathbb{R} \),
n cannot be negative, so \( n \geq 0 \).

\[
\sqrt{28} \ m^n \ + m^n \sqrt{63n} = \sqrt{4 \cdot 7 \cdot m^n \cdot n + m^n \cdot 9 \cdot 7 \cdot n} \\
= 2m^n \sqrt{7n} + 3m^n \sqrt{7n} \\
= 5m^n \sqrt{7n}
\]

c) \( 4\sqrt{2p^2q} - 6p\sqrt{2pq} \)

The cube root of a number is defined for all real numbers. So, each
radical is defined for \( p, q \in \mathbb{R} \).

\[
4\sqrt{2p^2q} - 6p\sqrt{2pq} = 4\sqrt{2p^2 \cdot pq} - 6p\sqrt{2pq} \\
= 4p \sqrt{2pq} - 6p\sqrt{2pq} \\
= -2p \sqrt{2pq}
\]

8. Simplify.

a) \( \sqrt{5b} + 4\sqrt{5b} - 3\sqrt{5b} - 2\sqrt{5b}, b \geq 0 \)

\[
= \sqrt{5b} + 4\sqrt{5b} - 2\sqrt{5b} - 3\sqrt{5b} \\
= 3\sqrt{5b} - 3\sqrt{5b}
\]

b) \( 3\sqrt{x^3} + 5\sqrt{2x} - \sqrt{4x^3}, x \geq 0 \)

\[
= 3\sqrt{x^3 \cdot x} + 5\sqrt{2x} - \sqrt{4 \cdot x^2 \cdot x} \\
= 3x\sqrt{x} + 5\sqrt{2x} - 2x\sqrt{x} \\
= x\sqrt{x} + 5\sqrt{2x}
\]

c) \( 5e\sqrt{24e^2} + 7\sqrt{54e^2} + e^2\sqrt{6e} + 6e, e \geq 0 \)

\[
= 5e\sqrt{4 \cdot 6 \cdot e^2} + 7\sqrt{9 \cdot 6e} + e^2\sqrt{6e} + 6e \\
= 5e(2e)\sqrt{6e} - 7(3e^2)\sqrt{6e} + e^2\sqrt{6e} + 6e \\
= 10e^2\sqrt{6e} - 21e\sqrt{6e} + e^2\sqrt{6e} + 6e \\
= -10e^2\sqrt{6e} + 6e
\]

d) \( \sqrt{16v^3} + \sqrt{3w^3} + 2w\sqrt{24w} - 5v\sqrt{54v}, v, w \in \mathbb{R} \)

\[
= \sqrt{8 \cdot 2 \cdot v^2 \cdot v} + \sqrt{3 \cdot w^2 \cdot w} + 2w\sqrt{8 \cdot 3 \cdot w} - 5v\sqrt{27 \cdot 2 \cdot v^2} \\
= 2v\sqrt{2v} + \sqrt{3w} + 2w(2\sqrt{2})\sqrt{3w} - 5v(3\sqrt{3} \cdot \sqrt{2v}) \\
= 2v\sqrt{2v} + \sqrt{3w} + 4w\sqrt{3w} - 15v\sqrt{2v} \\
= -13v\sqrt{2v} + 5w\sqrt{3w}
\]
9. A square with area 24 square units is placed
beside a square with area 50 square units.
In simplest form, write a radical expression for the perimeter of the shape formed.

The side length of a square is the square root of its area.
Small square:
Its area is 24, so its side length is \( \sqrt{24} \).
Large square:
Its area is 50, so its side length is \( \sqrt{50} \).
The perimeter of the shape formed consists of 3 sides of each square and the length that is the difference in their side lengths.
Perimeter of shape formed
\[
= 3 \sqrt{24} + 3 \sqrt{50} + (\sqrt{50} - \sqrt{24})
= 3 \sqrt{24} + 3 \sqrt{50} + \sqrt{50} - \sqrt{24}
= 2 \sqrt{24} + 4 \sqrt{50}
= 2 \sqrt{6 \cdot 4} + 4 \sqrt{50 - 2}
= 4 \sqrt{6} + 20 \sqrt{2}
\]

10. Two squares are enclosed in a large square
as shown. The area of the smallest square
is \( A \) square units. The area of the middle
square is \( 4A \) square units. Determine the
area and perimeter of the shaded region
in terms of \( A \).

The side length of a square is the square root of its area.
So, the side length of the small square is: \( \sqrt{A} \) units
The side length of the middle square is: \( \sqrt{4A} \), or \( 2\sqrt{A} \) units
The side length of the large square is the sum of the side lengths of the other 2 squares: \( \sqrt{A} + 2\sqrt{A} \), or \( 3\sqrt{A} \)
Area of shaded region
\[
= \text{area of large square} - \text{area of small square} - \text{area of middle square}
= (3\sqrt{A})^2 - A - 4A
= 9A - 5A
= 4A
\]
From the diagram, the length of 2 grid squares is \( \sqrt{A} \).
The perimeter of the shaded region is the length of 20 grid squares.
So, perimeter = 20 grid squares
\[
= 10 \cdot (2 \text{ grid squares})
= 10\sqrt{A}
\]
11. In right \( \triangle ABC \), \( AB \) has length 3 units and \( AC \) has length 6 units. A congruent triangle is placed adjacent to \( \triangle ABC \) as shown. Determine the perimeter of the shape formed.

Use the Pythagorean Theorem in \( \triangle ABC \) to determine the length of \( BC \).

\[
(AC)^2 = (BC)^2 + (AB)^2
\]

\[
6^2 = (BC)^2 + 3^2
\]

\[
27 = (BC)^2
\]

\[
\sqrt{27} = BC
\]

\[
3\sqrt{3} = BC
\]

The perimeter of each triangle is: \( 6 + 3 + 3\sqrt{3} = 9 + 3\sqrt{3} \)

\( BE = AB = 3 \)

So, the perimeter of the shape formed is:

\[
2 \times \text{perimeter of } \triangle ABC - 2 \times BE
\]

\[
= 2(9 + 3\sqrt{3}) - 2(3)
\]

\[
= 18 + 6\sqrt{3} - 6
\]

\[
= 12 + 6\sqrt{3}
\]

So, the perimeter of the shape formed is \( (12 + 6\sqrt{3}) \) units.

12. Determine whether \( \triangle EDG \) is a right triangle. How did you find out?

Use the Pythagorean Theorem in right \( \triangle DFG \) to determine the length of \( DF \).

\[
(DG)^2 = (FG)^2 + (DF)^2
\]

\[
(6\sqrt{3})^2 = (3\sqrt{12})^2 + (DF)^2
\]

\[
180 = 108 + (DF)^2
\]

\[
180 = 108 + (DF)^2
\]

\[
DF = \sqrt{72}
\]

Use the Pythagorean Theorem in right \( \triangle DEF \) to determine the length of \( ED \).

\[
(ED)^2 = (EF)^2 + (DF)^2
\]

\[
(ED)^2 = (11\sqrt{3} - 3\sqrt{12})^2 + (6\sqrt{2})^2
\]

\[
180 = 75 + 72
\]

\[
(ED)^2 = 147
\]

\[
ED = \sqrt{147}
\]

To determine whether \( \triangle EDG \) is a right triangle, use the Pythagorean Theorem to check whether \( (EG)^2 = (ED)^2 + (DG)^2 \).

\[
L.S. = (EG)^2 \quad R.S. = (ED)^2 + (DG)^2
\]

\[
= (11\sqrt{3})^2 \quad = (7\sqrt{3})^2 + (6\sqrt{5})^2
\]

\[
= 363 \quad = 147 + 180
\]

\[
= 327
\]

Since \( L.S. \neq R.S. \), \( \triangle EDG \) is not a right triangle.
13. Determine if there are any values of \( x \) and \( y \) such that \( \sqrt{x} + y \) and \( \sqrt{x} + \sqrt{y} \) are equal. Explain your reasoning.

\[ x, y \geq 0 \]
\[ \sqrt{x} + y = \sqrt{x} + \sqrt{y} \]
\[ (\sqrt{x} + y)^2 = (\sqrt{x} + \sqrt{y})^2 \]
\[ x + y = x + 2\sqrt{xy} + y \]
\[ 2\sqrt{xy} = 0 \]

For \( xy = 0 \), \( x = 0 \), or \( y = 0 \), or both \( x = 0 \) and \( y = 0 \).
So, there are values of \( x \) and \( y \) such that \( \sqrt{x} + y = \sqrt{x} + \sqrt{y} \).